**MAE210A: Fluid Mechanics I** Fall Quarter 2005
http://maecourses.ucsd.edu/mae210a

# Homework 0

For practice. Do not hand in. Stars indicate more challenging problems.

**1** Find a vector $\mathbf{x}$ for which $\mathbf{u}(\mathbf{x}\cdot\mathbf{u}) + \mathbf{x} = \mathbf{v}$ where $\mathbf{u}$ and $\mathbf{v}$ are given vectors.

**2** Find the locations of the minima of the function $f(x, y) = 2x^2 + y^2 - 3y + 2$ in the plane. Now do the same in the circle of unit radius centered at the origin.

**3** Calculate

- $\nabla^2 r^{-n}$
- $\nabla(\Omega\cdot\mathbf{x})$ where $\Omega$ is a constant vector
- $\nabla f$, where $f$ is the scalar function $f \equiv A_{ij}x_i x_j + B_j x_j + C$. What happens if $A$ is a symmetric matrix?

**4** Compute the surface integral $-\int_S p\,\mathrm{d}\mathbf{S}$ where the integrand is given by $p(z) = p_0 + N^2 z^2/2g$, where $S$ is a closed surface enclosing a volume $V$. [Hint: think about the location of center of mass of the volume.]

**5*** If the vector field $\mathbf{u}(\mathbf{x})$ is irrotational, show that for a closed volume

$$\int_V |\mathbf{u}|^2\,\mathrm{d}\mathbf{x} = \int_S \phi\frac{\partial\phi}{\partial n}\,\mathrm{d}S.$$

What is $\phi$ in this expression? [Optional: what happens if $V$ is unbounded?]

**6*** Calculate the integral

$$\int_V x_i x_j x_k\,\mathrm{d}V$$

over a sphere of radius $a$. [Hint: think about isotropic third-rank tensors.]

**7** Find the principal axes of the tensor field $t_{ij} = x_i x_j - \delta_{ij} r$ at the point $(2, 1, -1)$.