# Homework 1

Due Oct 3, 2006.

**1** Show using suffices that

$$\mathbf{u} \times (\nabla \times \mathbf{u}) = \nabla(\tfrac{1}{2}\mathbf{u}\cdot\mathbf{u}) - (\mathbf{u}\cdot\nabla)\mathbf{u}.$$

For what kinds of velocity fields does the left-hand side vanish?

**2** [Lautrup 10.2] A displacement field is given by

$$\mathbf{u} = [\alpha(x+2y) + \beta x^2, \alpha(y+2z) + \beta y^2, \alpha(z+2x) + \beta z^2].$$

Compute the divergence and curl of this field. Calculate the strain tensor

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).$$

**3** [210A midterm F2005] Consider the velocity field

$$\mathbf{u} = \left(-y, x, \frac{1}{x^2+y^2+t+1}\right).$$

At $t = 2\pi$, compute the streamlines and the streakline made up of dye released from $(1,0,0)$ during $0 \leq t \leq 2\pi$. Compute the particle path starting at $(1,0,0)$ at $t = 0$. [You may leave the results in parametric form.]

**4** [Kundu & Cohen 3.11] A flow field on the $xy$-plane has velocity components

$$u = 3x + y, \qquad v = 2x - 3y.$$

Show that the circulation around the circle $(x-1)^2 + (y-6)^2 = 4$ is $4\pi$.