# Homework 5

Due Nov 28, 2006.

**1** Water flow down a vertical circular pipe under gravity. The pipe walls are pulled up with velocity $U$. Is it possible for there to be zero net volume flux down the pipe? If so, what is $U$?

**2** A circular cylinder of radius $a$ rotates with angular velocity $\Omega \cos \omega t$. Compute the flow of a viscous fluid inside the cylinder. [You may want to ignore transients.]

**3** [Acheson 2.14] Consider in $y \geq 0$ the 2-D flow

$$u = \alpha x f'(\eta), \qquad v = -(\nu\alpha)^{1/2} f(\eta),$$

where

$$\eta = (\alpha/\nu)^{1/2} y.$$

Show that it is an exact solution of the Navier–Stokes equations which (i) satisfies the boundary conditions at the stationary rigid boundary $y = 0$ and (ii) takes the asymptotic form $u \sim \alpha x$, $v \sim -\alpha y$ far from the boundary if

$$f''' + f f'' + 1 - f'^2 = 0,$$

with

$$f(0) = f'(0) = 0, \qquad f'(\infty) = 1.$$

**Bonus:** solve the equation numerically and plot your solution. Show that $f'(3) = 0.998$.

**4** Find the dimensionless parameters for flow of a thin film of depth $h$ with surface tension $\sigma$. [You should find at least Froude, Weber and Reynolds numbers.]