Midterm

This is an 80 minute closed-book closed-note exam. No calculators.

1 (30 points) Fluid flows between two parallel plates at $y = -h$ and $y = h$. The lower plate has velocity $-U$ and the upper plate has velocity U.

(i) Show that the velocity field $\mathbf{u} = (Uy/h, 0, 0)$ satisfies the incompressible Navier–Stokes equations.

(ii) What is the pressure gradient? What is the shear stress?

(iii) Write down the integral form of the momentum balance, evaluating the terms.

(iv) Write down the different terms in the mechanical energy equation. Which terms balance? Write down the integral form of the equation, evaluating the terms.

2 (40 points) The streamfunction for inviscid flow inside the two-dimensional ellipse with boundary $x^2/a^2 + y^2/b^2 = 1$ is given by $\psi = -A(t)(x^2/a^2 + y^2/b^2)$.

(i) Show that the no-normal flow condition at the boundary is satisfied.

(ii) Compute the vorticity ω and the rate of strain tensor e_{ij}.

(iii) Compute the circulation around the boundary of the ellipse.

(iv) Show that there is a body force if $\dot{A} \neq 0$. Find the pressure gradient when $\dot{A} = 0$.

(iv) This flow is unsteady. For this flow, do particle paths coincide with streamlines?

3 (30 points) A water clock is an axisymmetric tank with profile $r = h(z)$. Determine the function $h(z)$ so that the downward velocity of the water surface is constant in time.