MAE210A: Fluid Mechanics I Winter Quarter 2018
http://web.eng.ucsd.edu/~sgls/MAE210A_2018

Final

This is a three hour open-book exam. Answer all six questions. The questions are all worth the same number of points.

1 Consider the velocity field written in suffix notation as

$$u_i = a_{ij} x_j$$

where the quantities a_{ij} are constants. Find the conditions on the constants for the velocity field to be (i) incompressible, (ii) irrotational. Show that the second condition can be written as $a_{ij} = a_{ji}$ and explain how this is consistent with the decomposition of velocity gradients discussed in class. Find the velocity potential when the flow is irrotational (but not necessarily incompressible).

2 For the velocity field

$$\boldsymbol{u} = [(t + \cos t)y, 0, -z],$$

compute the particle path starting from $(1, 1, 1)$ at $t = 0$, the streakline at $t = \pi$ made up of dye released from $(1, 1, 1)$ during $0 \leq t \leq \pi$, and the streamlines at $t = 0$. Results can be left in parametric form.

3 A water clock is an axisymmetric tank with profile $r = h(z)$. Determine the function $h(z)$ so that the downward velocity of the water surface is constant in time.

4 Newtonian fluid flows between two concentric tubes with $a < r < b$ with no pressure gradient. The inner tube moves at velocity U while the outer tube is at rest. Compute the fully-developed velocity field, volume flux and average velocity. What is the force on the boundary over the region $0 < x < L$?

5 Sketch the streamlines represented by $w = Az^2$ (A is real), and show that the speed is everywhere proportional to the distance from the origin. Compute the fluxes of mass and momentum through the four sides of the square $0 < x < l$, $0 < y < l$ (take constant density).

6 Surface gravity waves can be described by an angular frequency ω and wavelength λ. What are the units of ω and λ? The physical parameters governing the propagation of these waves are h, the depth of the water, g, the acceleration due to gravity, T, the surface tension of the air-water interface surface tension (units: force per unit length), ρ, the density of the water, and μ, the dynamic viscosity of the water. Use dimensional analysis to obtain the simplest version of the dispersion relation relating ω to λ.