Homework VI

Due Mar 2, 2018.

1 Consider the complex potential

$$w = z^2 + \frac{m}{2\pi}\log z$$

where m is real. Write down the velocity potential ϕ and streamfunction ψ. Show that the x- and y-axes are streamlines. What is the flux of fluid through the circle with radius a? What is the circulation around the same circle? [Hint: you may find it easiest to use polar coordinates.]

2 Sketch the streamlines corresponding to the complex potential $w = \cosh^{-1}(z/c)$. Show that a limiting case is the flow through an aperture of width $2c$ in an otherwise infinite flat plate. Calculate the velocity of the fluid at the edges of the aperture.

3 A circular cylinder moves along the x-axis with velocity $U(t)$. There is no circulation about the circle. Show that the instantaneous complex potential when the circle is centered at the origin is $w = -Ua^2/z$. Hence show that the kinetic energy of the fluid is $\frac{1}{2}\pi a^2 \rho U^2$. Deduce using energy considerations the governing equation of the motion of the cylinder in the form

$$M\frac{\mathrm{d}U}{\mathrm{d}t} = F - M'\frac{\mathrm{d}U}{\mathrm{d}t},$$

where M is the mass of the cylinder per unit length and F is the external force per unit length. What is M'? How else might you deduce this result?

4 [Kundu 6.5] A two-dimensional potential vortex with clockwise circulation Γ is located at point $(0, a)$ above a flat plate. The plate coincides with the $x-$axis. A uniform stream U directed along the x-axis flows over the vortex. Sketch the flow pattern and show that it represents flow over an oval-shaped body. [*Hint:* Introduce the image vortex and locate the two stagnation points on the x-axis.] If the pressure at $x = \pm\infty$ is p_∞, and that *below* the plate is also p_∞, then show that the pressure at any point on the plate is given by

$$p_\infty - p = \frac{\rho\Gamma^2 a^2}{2\pi^2(x^2+a^2)^2} - \frac{\rho U\Gamma a}{\pi(x^2+a^2)}.$$

Show that the total upward force on the plate is

$$F = \frac{\rho\Gamma^2}{4\pi a} - \rho U\Gamma.$$