Homework VII

Due Mar 9, 2018.

1 A long circular tube has a layer of liquid of uniform thickness adhering to its inner surface. In order to remove the liquid, air is blown through the tube by applying a pressure difference between the two ends. Determine the ratio of the steady volume fluxes of the air and the liquid.

2 Two incompressible fluids of the same density ρ and dynamic viscosities μ_B and μ_T flow, one of top of the other, down an inclined plane. Determine the velocity profiles in the two layers, which are of depth h_B and h_T. The velocity profile in the bottom layer depends on h_T but not on μ_T. Why?

3 [Kundu 9.5] Suppose a line vortex of circulation Γ is suddenly introduced into a fluid at rest. Show that the solution is

$$u_\theta = \frac{\Gamma}{2\pi r} \mathrm{e}^{-r^2/4\nu t}.$$

Sketch the velocity distribution at different times. Calculate and plot the vorticity, and observe how it diffuses outward. [See § 9.9 for a similar problem and approach.]

4 [Acheson 2.5] Viscous fluid is at rest in a two-dimensional channel between two stationary rigid walls $y = \pm h$. For $t \geq 0$ a constant pressure gradient $P = -\mathrm{d}p/\mathrm{d}x$ is imposed. Show that $u(y,t)$ satisfies

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{P}{\rho},$$

and give suitable initial and boundary conditions. Find $u(y,t)$ in the form of a Fourier series, and show that the flow approximates to steady channel flow when $t \gg h^2/\nu$.