MAE224A: Environmental Fluid Mechanics Fall Quarter 2010
http://maecourses.ucsd.edu/ sllewell/MAE224A_2010/

Midterm

This is a 50 minute open-book exam (no books). Answer all three questions. Explain your working and state any assumptions you have made.

1 (5 points) Answer briefly (a few sentences):

1. Discuss the form of the conversion term from kinetic energy to potential energy. Does such a term always exist?
2. Pick two non-dimensional numbers; discuss how they can be obtained and their physical significance.
3. Find the power law solution to the two-dimensional plume equations in an unstratified medium

$$\frac{\mathrm{d}Q}{\mathrm{d}z} = 2\alpha\frac{M}{Q}, \qquad \frac{\mathrm{d}M}{\mathrm{d}z} = \frac{BQ}{M}, \qquad \frac{\mathrm{d}B}{\mathrm{d}z} = -N^2 Q.$$

2 (6 points) In Rayleigh–Bénard convection, two plates a distance H apart are maintained at a temperature difference ΔT Starting from the Boussinesq Navier–Stokes equations with the linearized equation of state

$$\rho = \rho_0[1 - \beta(T - T_0)],$$

obtain non-dimensional equations and show that these depend on two non-dimensional parameters, conventionally taken to be

$$R = \frac{g\beta\Delta T H^3}{\kappa\nu}, \qquad \sigma = \frac{\nu}{\kappa},$$

where ν is the kinematic viscosity of the fluid and κ is the heat diffusion coefficient. What are the units of β? Will the flow be turbulent for large or small R? Why?

3 (9 points) In Rayleigh-Bénard convection, the motion for large R is observed to be turbulent, with two thin boundary layers separating the bulk of the fluid which is at a mean temperature $T_0 + \frac{1}{2}\Delta T$, where T_0 is the temperature of the cooler plate. Thermals rise from the boundary layer as blobs of light fluid (and similarly with descending blobs of fluid). A heuristic theory suggests that these plumes break away when the local Rayleigh number computed using the temperature difference between the lower plate and the interior and using the boundary layer thickness δ is around 1800.

(i) Show that an appropriate Rayleigh number for a blob about to leave the boundary layer is

$$R_\delta = \frac{g'\delta^3}{\kappa\nu}.$$

where $g' = \frac{1}{2}g\beta\Delta T$.
(ii) Find δ/H.
(iii) Taking the blob as a thermal, show that the initial values of the variables are

$$V_0 = \delta^3, \qquad M_0 = 0, \qquad B_0 = g'\delta^3.$$

(iv) Integrate the thermal equations from $z = \delta$ to $z = H - \delta$. What is the final size of the blob? What is its final temperature? Is this consistent with the setup of the problem?