Homework 0

For review. Do not hand in.

1 Find the real and imaginary parts of the following complex numbers:

$$(a) \quad z_1 = \frac{e^{-2i\theta} + 2}{e^{2i\theta} - 2}, \qquad (b) \quad z_1 = \frac{-2 + i}{1 - i}, \qquad (c) \quad z_3 = \left(\frac{1 + i\sqrt{3}}{2} \right)^{2016}.$$

2 Solve the following problems

$$\begin{array}{lll} (a) & y'' + y = e^x, & y(0) = y'(0) = 0, \\ (b) & y' - x^2 y = \sin x, & y(0) = 0, \\ (c) & yy'' + y'^2 = 2, & y(0) = y'(0) = 0, \\ (d) & y' = (1+y)^{-1}, & y(0) = 0. \end{array}$$

3 Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

4 Sketch the graph of $x^{-1} \tanh x$ over the real axis.

5 Use Euler's formula to express $\sin 4x$ in terms of $\sin x$, $\cos x$ and powers of those functions.

6 Show that if $g(x)$ is a solution of the nonlinear ODE

$$g''' + gg'' = 0,$$

then so is $f(x) = \lambda g(\lambda x)$. Explain how to obtain the solution to the boundary-value problem with boundary conditions $f(0) = f'(0) = 0$, $f'(\infty) = 1$ (*) from the solution to the initial-value problem with boundary conditions $f(0) = f'(0) = 0$, $f''(0) = 1$ (**). Use the result that as $x \to \infty$, the solution to the initial-value problem (**) has $f' \to \mu$.
[Bonus: do this numerically. You should find that for (*), $f''(0) = 0.4969\ldots$; obtain 12 digits.]