

# Homework I

Due Oct 12, 2018.

- 1 Find the general solution to

$$y''' + y = 0.$$

- 2 Find the solution to

$$y'' + 4y = 0, \quad y(0) = 0, \quad y(1) = 1.$$

- 3 Find the general solution to

$$x^3 y''' + 3x^2 y'' + xy' + y = 0.$$

- 4 Find the solution to

$$x^2 y'' + 5xy' + 5y = 5, \quad y(1) = 1, \quad y'(1) = 1.$$

- 5 Find the general solution to

$$y' + e^{-x}y = \sin x.$$

- 6 Analysis of Stokes flow past a two-dimensional cylinder yields the ODE

$$\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} \right)^2 f = 0.$$

Find the general solution. The appropriate boundary conditions are that  $f$  must grow no faster than  $r$  for large  $r$  and that  $f(a) = f'(a) = 0$ . Explain why there is a problem. This is *Stokes' paradox*.