# Homework II

Due Oct 19, 2018.

**1** Find the general solution to

$$y'' + (\log x)y' - (1 + \log x)y = 0.$$

**2** Find a simple solution to the problem

$$y'' + p(x)[xy' - y] = 0.$$

Use reduction of order to obtain a second solution. Find the solution explicitly when $p(x) = \cos x$. Show that, with boundary conditions $y(\pi) = 0$ and $y(2\pi) = 1$, the solution is

$$y(x) = \frac{x \displaystyle\int_\pi^x a^{-2} e^{-a \sin a - \cos a}\, da}{2\pi \displaystyle\int_\pi^{2\pi} a^{-2} e^{-a \sin a - \cos a}\, da}.$$

**3** Solve the problem

$$y' + y \sin x = f(x), \qquad y(0) = 0$$

using a Green's function approach. Show that the same solution is found using an integrating factor.

**4** Solve the problem

$$y'' + 2y' + y = \frac{1}{1 + e^{2x}}, \qquad y(0) = 0, \quad \lim_{x \to \infty} e^x y(x) = 0$$

using a Green's function approach.

**5** Solve the equation

$$f'' + \frac{1}{r}f' - \frac{1}{r^2}f = g(r), \quad f(0) = 0$$

using a Green's function approach. You should have an unknown constant in your answer. Discuss this and its relation to the behavior of the solution for large $r$.

**6** Find the general solution to

$$y'' + (\operatorname{sgn} x)y = e^{-|x|}.$$