

## Homework IV

Due Nov 16, 2018.

- 1 Find the eigenvalues and eigenfunctions of the problem

$$y'' + \lambda \frac{y}{x^2} = 0; \quad y(1) = 0, \quad y(2) = 0.$$

Give numerical results for the two lowest eigenvalues. Can you find an approximate result for  $\lambda_n$  for large  $n$ ?

- 2 Use the Rayleigh quotient to find the lowest eigenvalue of the problem.

$$y'' + \lambda e^x y = 0; \quad y(0) = 0, \quad y(1) = 0.$$

Try the following test function:  $y = \sin \pi x + a \sin 2\pi x$ . The exact result is 5.82654627418.

- 3 Find an exact solution to

$$y' = y^2 - y \sin x + \cos x$$

and hence find a second solution.

- 4 Solve the equation

$$y' = -\frac{xe^{-y}}{x^2 + y^2}.$$

- 5 Solve the equation

$$y'' + \frac{1}{x}y' + \frac{y^2}{x^2} = 0.$$

- 6 Solve the equation

$$y' - x \frac{y^2}{y + y'} = 0.$$