Homework V

Due Nov 30, 2018.

1   Find the most general solutions $u(x,y)$ to the following equations, consistent with the boundary conditions stated:

$$\text{(a)} \qquad (y-2)\frac{\partial u}{\partial x} + (x+1)\frac{\partial u}{\partial y} \;=\; 0, \qquad u(x,0) = x,$$

$$\text{(b)} \qquad e^{y}\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \;=\; 0, \qquad u(x,0) = 1-x,$$

$$\text{(c)} \qquad \sin y\,\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} \;=\; 0, \qquad u(x,0) = x^{2},$$

$$\text{(d)} \qquad \frac{\partial u}{\partial x} + 3x^{2}\frac{\partial u}{\partial y} \;=\; 0, \qquad u = 1 \text{ on the curve } y = x.$$

2   Find the most general solutions $u(x,y)$ to the following equations, consistent with the boundary conditions stated:

$$\text{(a)} \qquad (y-2)\frac{\partial u}{\partial x} + (x+1)\frac{\partial u}{\partial y} \;=\; y-2, \qquad u(x,0) = x,$$

$$\text{(b)} \qquad e^{y}\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \;=\; yu, \qquad u(x,0) = 1-x,$$

$$\text{(c)} \qquad \sin y\,\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} \;=\; x\sin y, \qquad u(x,0) = x^{2},$$

$$\text{(d)} \qquad \frac{\partial u}{\partial x} + 3x^{2}\frac{\partial u}{\partial y} \;=\; x^{2}, \qquad u = 1 \text{ on the curve } y = x.$$

3   Find the most general solutions $u(x,y)$ to

$$(2xy + \sin y)\frac{\partial u}{\partial x} + (e^{x} - y^{2})\frac{\partial u}{\partial y} = 0$$

subject to $u = x$ on $y = 0$.

4 Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y^2} = 2,$$

subject to $u = 3x$ and $\partial u/\partial y = 2$ on the line $y = 0$.

5 Solve

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} = 0,$$

subject to $u = 3$ and $\partial u/\partial x = 1$ on the line $y = x$.

6 Find a formal solution to

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + 2\frac{\partial^2 u}{\partial y^2} = 0.$$