Homework VI

Due Dec 7, 2018.

1 Find the solution $u(x,y)$ to Poisson's equation $\nabla^2 u = xy$ inside the square $0 < x < a$, $0 < y < b$ with u vanishing on the boundary.

2 Solve Helmholtz's equation $\nabla^2 u + k^2 u = 0$ inside the rectangle with boundaries $x = \pm a$, $y = \pm b$ and with boundary conditions $u = 1$ on $x = -a$ and $u = 0$ everywhere else.

3 Find the displacement $u(r,\theta,t)$ of a membrane occupying $0 < r < a$ that satisfies the wave equation with wavespeed c, clamped boundary at $r = a$ so that $u(a,\theta) = 0$ and initial conditions $u = 0$, and $u_t = \mathrm{e}^{-r^2} \sin 2\theta$.

4 Solve Laplace's equation for u inside the semicircle $r < a$, with boundary conditions $u = 0$ on $y = 0$ and $u = \theta(\pi - \theta)$ on $r = a$.

5 Solve the diffusion equation for T inside the infinite cylinder $r < a$, $z > 0$ with $T = 0$ on the boundary and $T = \mathrm{e}^{-z}(a^2 - r^2)$ at $t = 0$.

6 Solve the diffusion equation inside a sphere of radius R with $T = \mathrm{e}^{-\mathrm{i}\omega t} \cos\theta$ on the boundary, where θ is colatitude.