Homework II

Due Jan 23, 2020.

1 Show that the origin is a fixed point of the system

$$\begin{aligned}\dot{x} &= (x^2+y^2)\left[\frac{x}{2x^2+y^2}(1-ax^2-ay^2)-\frac{y}{x^2+2y^2}\right],\\ \dot{y} &= (x^2+y^2)\left[\frac{y}{2x^2+y^2}(1-ax^2-ay^2)+\frac{x}{x^2+2y^2}\right],\end{aligned}$$

and discuss its nature. For $a > 0$ show that $\dot{r} > 0$ for small r and $\dot{r} < 0$ for large r. The Poincaré–Bendixson theorem shows that there is then a periodic orbit; explain. Now transform to polar coordinates, find the periodic orbit and solve for $r(\theta)$.

2 Describe the bifurcation diagram for the system

$$\dot{x} = x(4-\mu x)(\mu+2x-x^2)(\mu^2-4\mu+x^2+2x+1)$$

showing the locus and stability of stationary points in the (x, μ) plane.

3 The Lorenz equations are

$$\begin{aligned}\dot{u} &= 10(v-u),\\ \dot{v} &= u(28-w)-v,\\ \dot{w} &= uv-(8/3)w\end{aligned}$$

for an interesting choice of parameter values. Find the fixed points. Now investigate the linear dynamics of this three-dimensional system about the fixed points. You should be able to generalize the two-dimensional analysis from class and interpret the resulting local dynamics.

4 The confluent hypergeometric function satisfies the equation

$$xy'' + (b-x)y' - ay = 0.$$

Obtain the controlling behavior of y near infinity. Discuss the dependence on a and b.