

Homework III

Due Jan 30, 2020.

- 1 Find the controlling behavior as $x \rightarrow \infty$ of the solutions to the equation

$$x^2 y'' + 2xy' + [x^2 - \nu(\nu + 1)]y = 0.$$

When $\nu = n$ is an integer, the solution that is regular at the origin has the integral representation

$$j_n(x) = \frac{x^n}{2^{n+1}n!} \int_0^\pi \cos(x \cos \theta) \sin^{2n+1} \theta \, d\theta.$$

Using this expression, find a closed form for $j_0(x)$. Show that this closed form is consistent with the controlling behavior that you obtained.

- 2 (Hinch 1.2) Find the rescalings for the roots of

$$\epsilon^2 x^3 + x^2 + 2x + \epsilon = 0$$

and thence find two terms in the approximation for each root.

- 3 Assume that the Earth is a perfect sphere of radius $R = 6400$ km and that it is wrapped tightly at the equator with a rope. Suppose one cuts the rope and splices a length $l = 1$ cm into the rope. Then the rope is grabbed at a point and hoisted above the surface of the Earth as high as possible. How high is that?

- 4 Find three terms in an approximation for the solution of

$$x^{-1} \exp(-x^2) = \epsilon.$$