Homework IV

Due Feb 27, 2020.

1 Find the values of α for which solutions to the equation

$$\ddot{x} + (1 + 4\epsilon \cos \alpha t)x = 0, \qquad 0 < \epsilon \ll 1$$

have increasing amplitude for $\epsilon t = O(1)$.

2 Obtain a solution valid for $\epsilon t = O(1)$ to the equation

$$\ddot{x} + x - \frac{\epsilon}{2 + \dot{x}} = 0,$$

with $x(0) = 0$, $\dot{x}(0) = 1$. You may find the following integral useful:

$$I = \int_0^{2\pi} \frac{\cos\theta}{2 + A\cos\theta}\,\mathrm{d}\theta = 2\pi \frac{\sqrt{4 - A^2} - 2}{A\sqrt{4 - A^2}}$$

for $0 < A < 2$. Bonus: obtain this integral.

3 (Kevorkian & Cole 4.3.5) Solve the boundary-value problem

$$y'' + y' - \epsilon y^2 = 0, \qquad y(0, \epsilon) = 0, \quad y(\epsilon^{-1}, \epsilon) = 1, \qquad 0 < \epsilon \ll 1$$

using (i) the method of multiple scales and (ii) boundary layer theory. Compare the two solutions.

4 (Johnson E2.13) Find (and match) two terms in the inner and outer expansions of the solution to

$$\epsilon y'' - y' + \epsilon x y^2 = 2x, \qquad y(0) = 2, \quad y(1) = 2 + \epsilon, \qquad 0 < \epsilon \ll 1.$$

[Hint: be careful to consider all the necessary terms in the matching, e.g. by using van Dyke's rule.]