Homework V

Due March 5, 2020.

1 Solve the Lighthill MAE example from class in the inner region using MAE.

2 Find two terms in the inner and outer solution of

$$\epsilon y'' + y'(x + \epsilon^2 y'^2) + y + \epsilon y^3 = 2x, \qquad y(0) = -1, \quad y(1) = 1$$

for small ϵ.

3 Consider the equation

$$\ddot{y} - \epsilon t \mathrm{e}^{-\epsilon t} y' + y = 0$$

where $0 < \epsilon \ll 1$. Change variable to $T = \epsilon t$ and solve the resulting equation using WKB. Do this with and without changing to Liouville normal form (the latter is a generalization of the usual WKB).

4 Find the large eigenvalues of the problem

$$-y'' + x^4 y = Ey$$

where y decays at $\pm\infty$. You may leave an unevaluated integral in the answer.