Midterm

This is an 80 minute open-note exam (no calculators, no books). Answer all four questions and justify your answers.

1 (Glendinning 5.6) Analyze the fixed points of the system

$$\dot{x} = x^2 - y - 1, \qquad \dot{y} = (x - 2)y.$$

Sketch the phase plane.

2 The algebraic form of Mathieu's equation is

$$(1 - \zeta^2)w'' - \zeta w' + (a + 2q - 4q\zeta^2)w = 0.$$

Find the controlling behavior of $w(\zeta)$ as $\zeta \to \infty$.

3 Find the first two terms in an expansion for small ϵ of the three roots of

$$\begin{array}{ll} (a) & \epsilon x^3 + x^2 + (2 + \epsilon)x + 1 = 0, \\ (b) & \epsilon x^3 + x^2 + (2 - \epsilon)x + 1 = 0. \end{array}$$

4 The Rayleigh oscillator is governed by the equation

$$\ddot{y} + \epsilon(\dot{y}^2 - 1)\dot{y} + y = 0$$

with $\epsilon > 0$. Use the method of multiple scales to show that the amplitude of the oscillation (twice the maximum value of $y(t)$) is $4/\sqrt{3}$ for small ϵ. For what initial conditions is this true?