Coded Modulation
An Information-Theoretic Perspective

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Information theory of point-to-point communication

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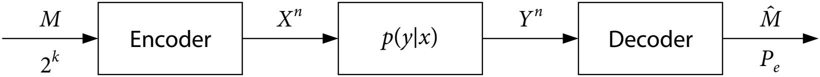

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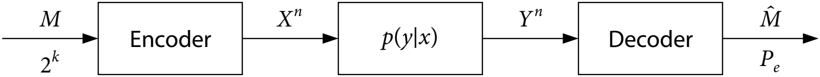

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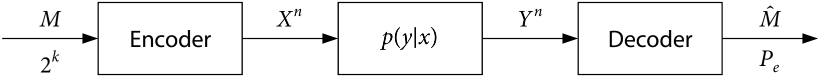

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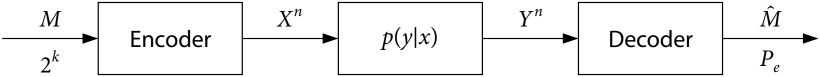

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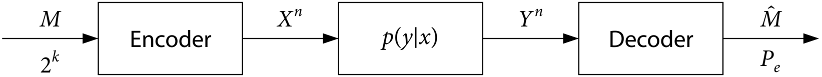

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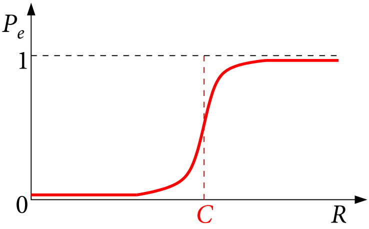

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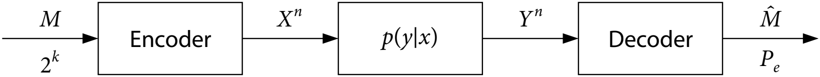

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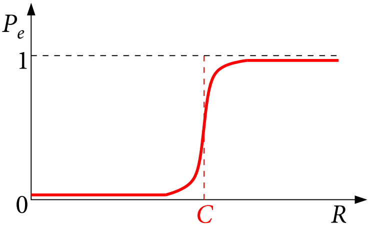

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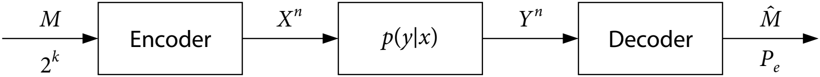

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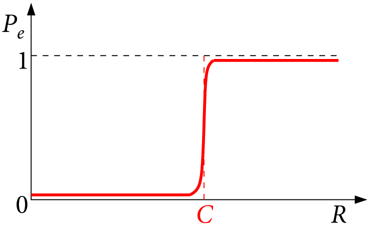

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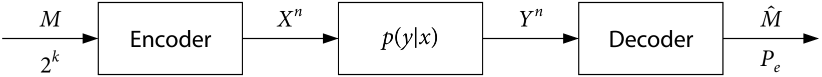

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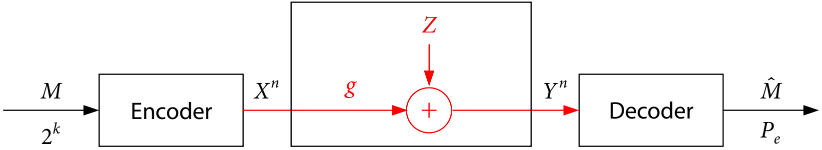
Channel coding theorem (Shannon 1948)

$$C = \max_{p(x)} I(X;Y)$$

Gaussian channel



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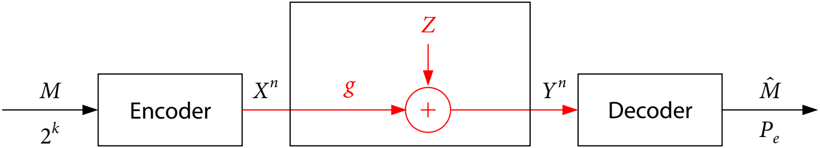

- Simple model for wireless, wired, and optical communication

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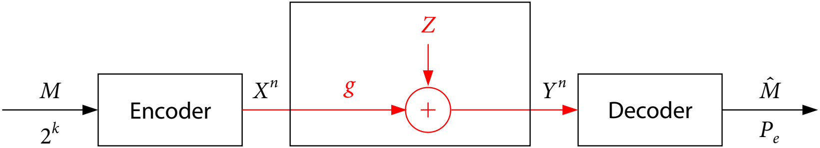

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- Average power constraint $\sum_{i=1}^{n} x_i^2(m) \le nP$

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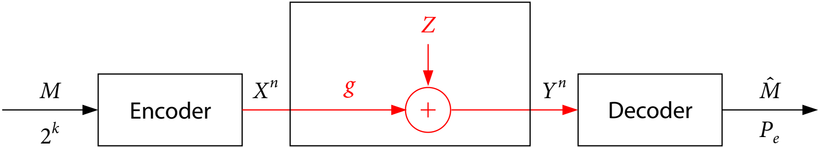

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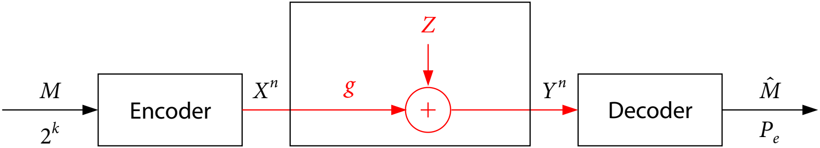

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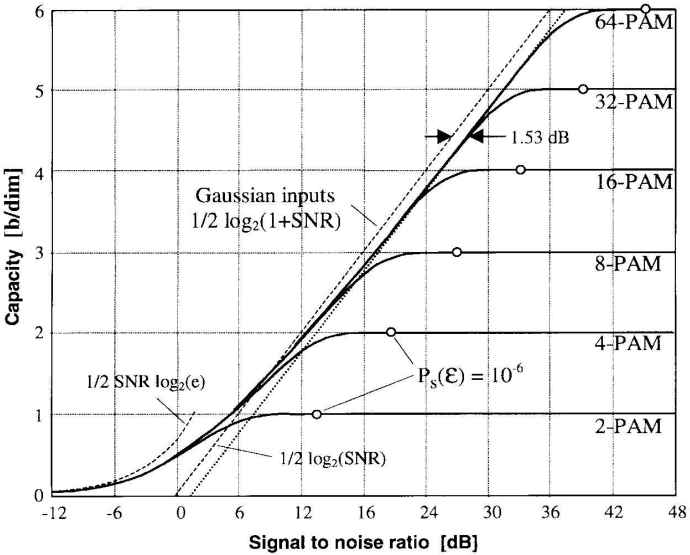
Capacity of the Gaussian channel (Forney–Ungerboeck '98)



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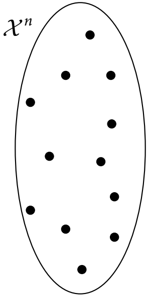

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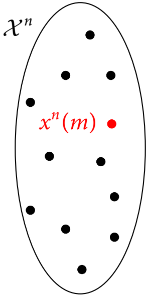

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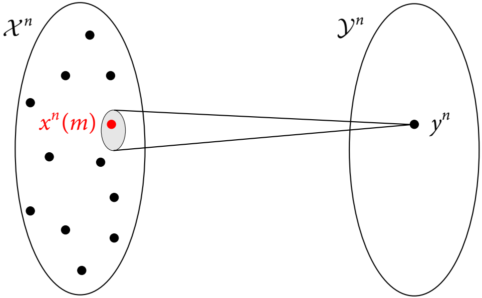

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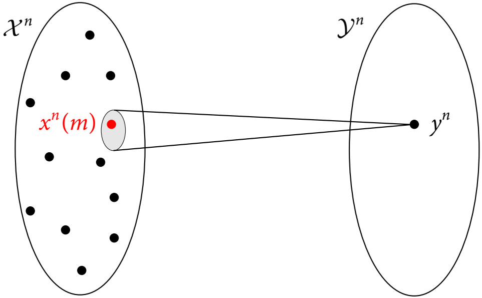

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- Successful w.h.p. if $R < I(X; Y)$

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- Average performance of randomly generate codes is good
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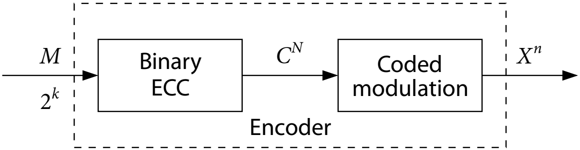
  - Everything is binary

Coded modulation



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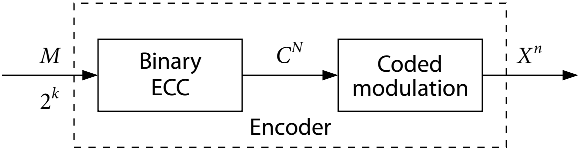

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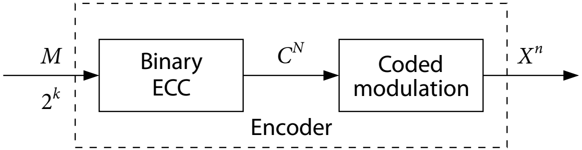

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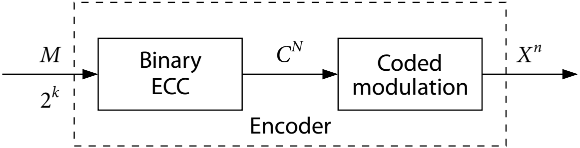

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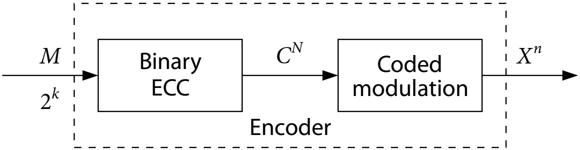

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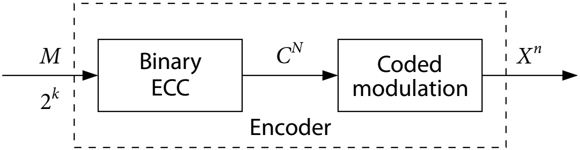

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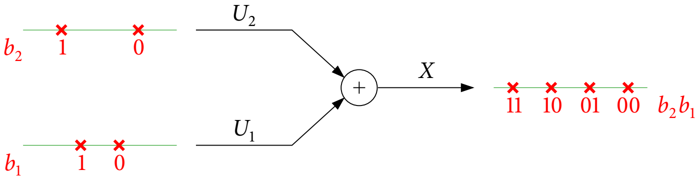
Multiple layers and symbol-level mapping



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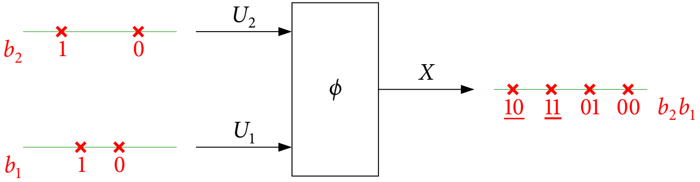

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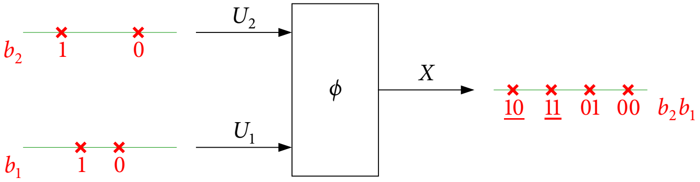

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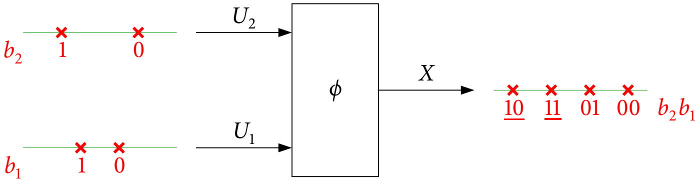

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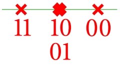
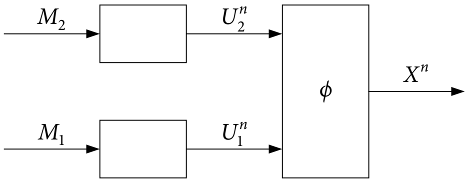
- Can be many-to-one (still information-lossless)



- Can induce nonuniform X (Gallager 1968)

Horizontal superposition coding



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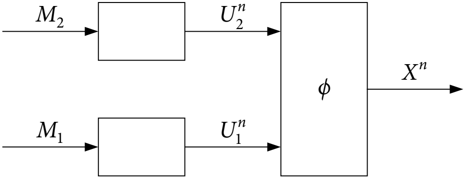

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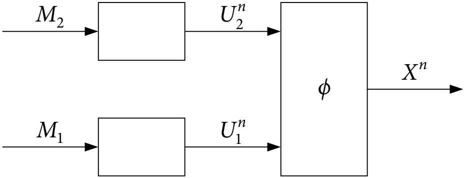

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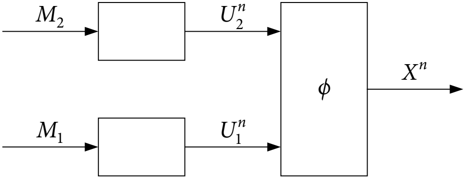

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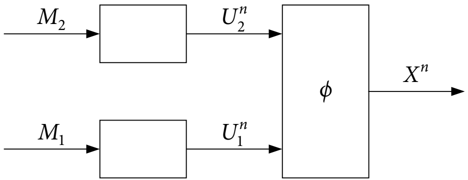

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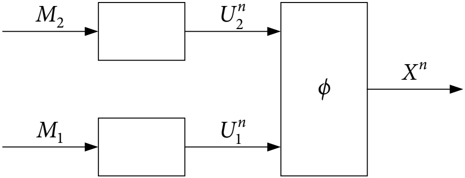

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$$R_1 + R_2 < I(U_1; Y, U_2) + I(U_2; Y)$$

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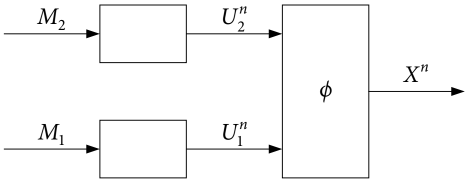

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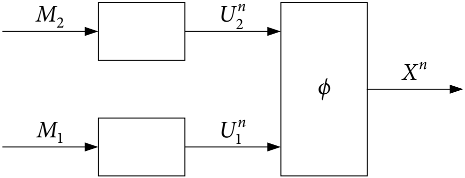

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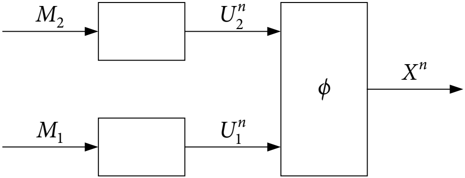

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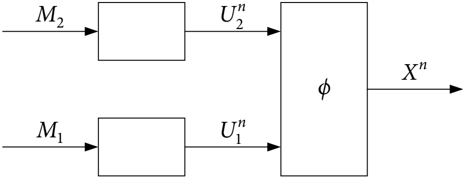
$$\begin{aligned} R_1 + R_2 &< I(U_1; Y, U_2) + I(U_2; Y) \\ &= I(U_1, U_2; Y) = I(X; Y) \end{aligned}$$

  - Regardless of ϕ or the decoding order

Horizontal superposition coding



- Broadcast channels (Cover 1972), fading channels (Shamai–Steiner 2003)
- Successive cancellation decoding:
  - Find a unique m_2 such that $(u_2^n(m_2), y^n)$ is jointly typical: $R_2 < I(U_2; Y)$
  - Find a unique m_1 such that $(u_1^n(m_1), u_2^n(m_2), y^n)$ is jointly typical: $R_1 < I(U_1; U_2, Y)$
  - Combined rate:

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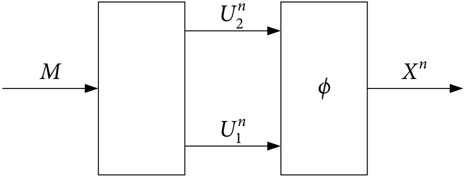
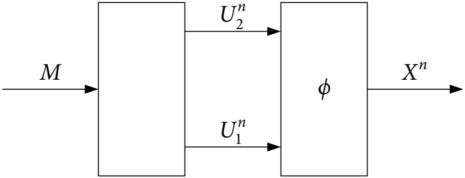
  - Regardless of ϕ or the decoding order
- Multi-level coding (MLC): Wachsmann–Fischer–Huber (1999)

Vertical superposition coding



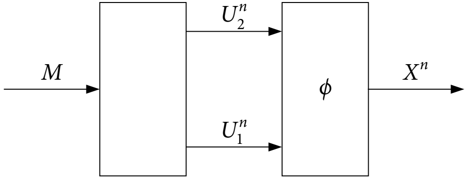
Vertical superposition coding



- Single codeword of length $2n$: $C^{2n} = (C^n, C_{n+1}^{2n})$

$$C^n \mapsto U_1^n \qquad C_{n+1}^{2n} \mapsto U_2^n$$

Vertical superposition coding



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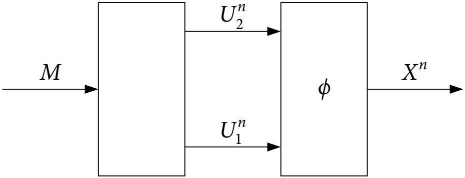
Vertical superposition coding



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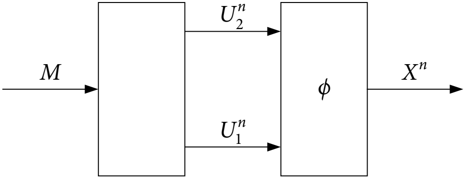
$$(u_1^n(m), y^n) \text{ is jointly typical} \quad \text{and} \quad (u_2^n(m), y^n) \text{ is jointly typical}$$

Vertical superposition coding



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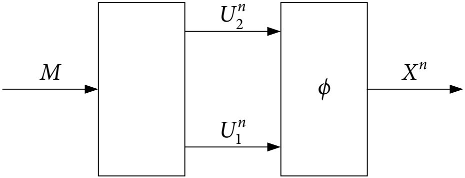
  - Successful w.h.p. if

$$R < I(U_1; Y) + I(U_2; Y)$$

Vertical superposition coding



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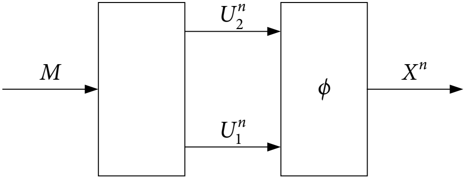

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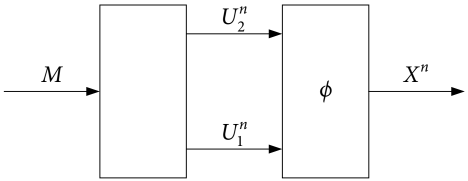
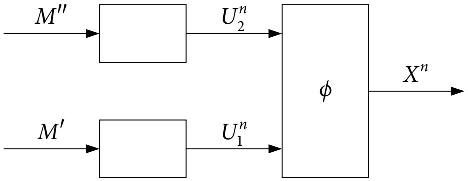
- Bit-interleaved coded modulation (BICM): Caire–Taricco–Biglieri (1998)

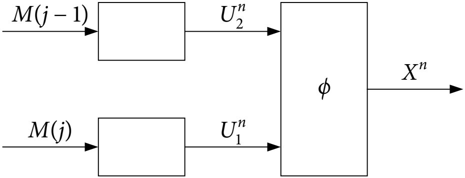
Diagonal superposition coding



Diagonal superposition coding



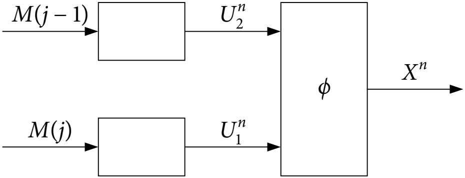
Diagonal superposition coding



- Think outside the block: Sequence of messages $M(j)$ mapped to $C^{2n}(j)$

Block	1	2	3	4	5	6	7
U_2							
U_1							

Diagonal superposition coding



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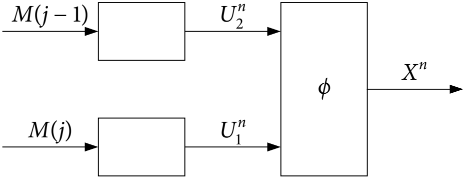
Block	1	2	3	4	5	6	7
U_2		$C_{n+1}^{2n}(1)$					
U_1	$C^n(1)$						

Diagonal superposition coding



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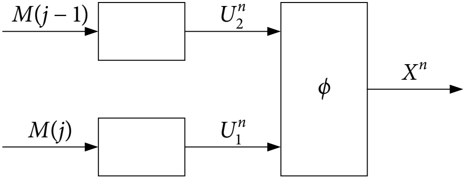
U_2		$C_{n+1}^{2n}(1)$	$C_{n+1}^{2n}(2)$				
U_1	$C^n(1)$	$C^n(2)$					

Diagonal superposition coding



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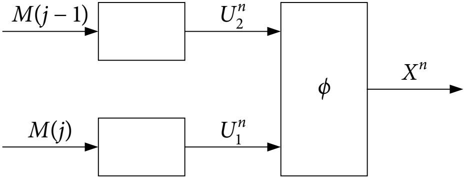

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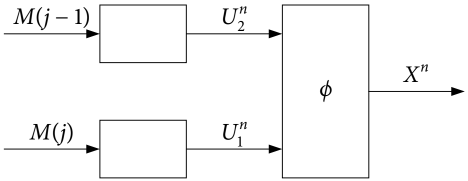
U_1	$C^n(1)$	$C^n(2)$	$C^n(3)$	$C^n(4)$			

Diagonal superposition coding



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U_2		$C^{2n}_{n+1}(1)$	$C^{2n}_{n+1}(2)$	$C^{2n}_{n+1}(3)$	$C^{2n}_{n+1}(4)$	$C^{2n}_{n+1}(5)$	
U_1	$C^n(1)$	$C^n(2)$	$C^n(3)$	$C^n(4)$	$C^n(5)$		

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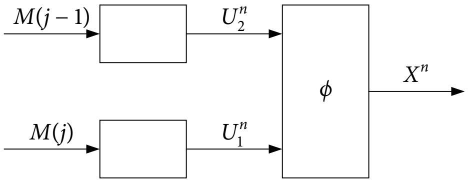

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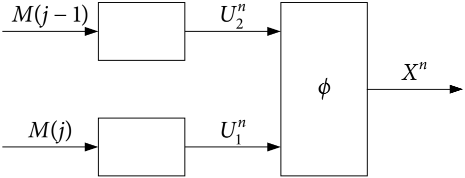

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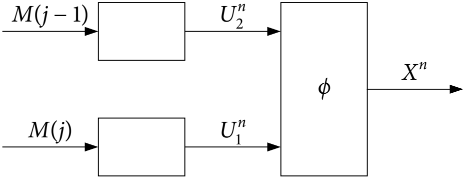
Diagonal superposition coding



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Block	1	2	3	4	5	6	7
U_2			$C_{n+1}^{2n}(2)$	$C_{n+1}^{2n}(3)$	$C_{n+1}^{2n}(4)$	$C_{n+1}^{2n}(5)$	$C_{n+1}^{2n}(6)$
U_1		$C^n(2)$	$C^n(3)$	$C^n(4)$	$C^n(5)$	$C^n(6)$	

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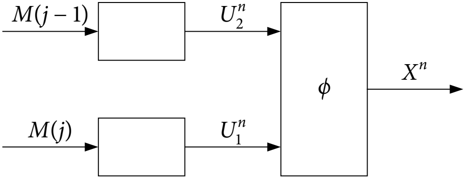

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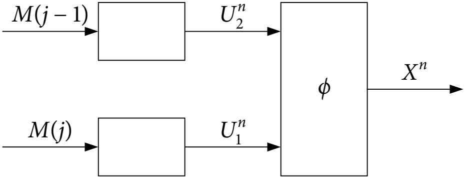

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Block	1	2	3	4	5	6	7
U_2			$C^{2n}_{n+1}(2)$	$C^{2n}_{n+1}(3)$	$C^{2n}_{n+1}(4)$	$C^{2n}_{n+1}(5)$	$C^{2n}_{n+1}(6)$
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- Block Markov coding: Used extensively in relay and feedback communication

- Sliding-window coded modulation (SWCM): Kim et al. (2016), Wang et al. (2017)

Multiple-antenna transmission

- Consider the signal layers U_1 and U_2 as antenna ports: $\mathbf{X} = (U_1, U_2)$

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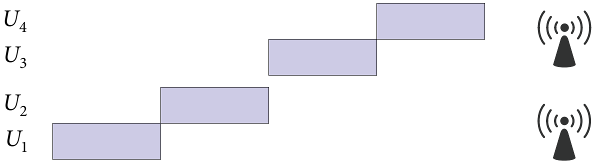
- Signal layers can be far more general than antenna ports
- Coded modulation can encompass MIMO transmission



Comparison

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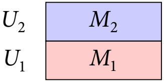
Horizontal



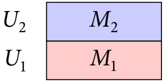
Multi-level coding (MLC)

$$R_2 < I(U_2; Y)$$
$$R_1 < I(U_1; U_2, Y)$$

Short, nonuniversal

Comparison

Horizontal



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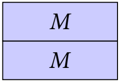
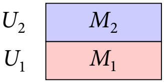
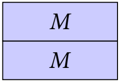
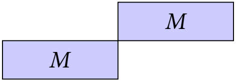


Bit-interleaved coded modulation (BICM)

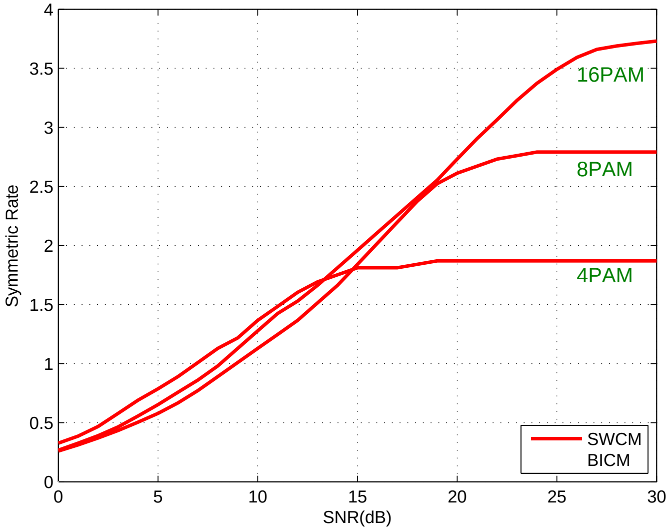
$$R < I(U_1; Y) + I(U_2; Y)$$

Other layers as noise

Comparison

Horizontal	Vertical	Diagonal
 	 	
Multi-level coding (MLC)	Bit-interleaved coded modulation (BICM)	Sliding-window coded modulation (SWCM)
$R_2 < I(U_2; Y)$ $R_1 < I(U_1; U_2, Y)$	$R < I(U_1; Y) + I(U_2; Y)$	$R < I(U_1; U_2, Y) + I(U_2; Y)$ $= I(X; Y)$
Short, nonuniversal	Other layers as noise	Error prop., rate loss

BICM vs. SWCM



LTE turbo code / ≤ 8-iteration LOG-MAP decoding at $b = 20$, $n = 2048$, BLER $= 0.1$

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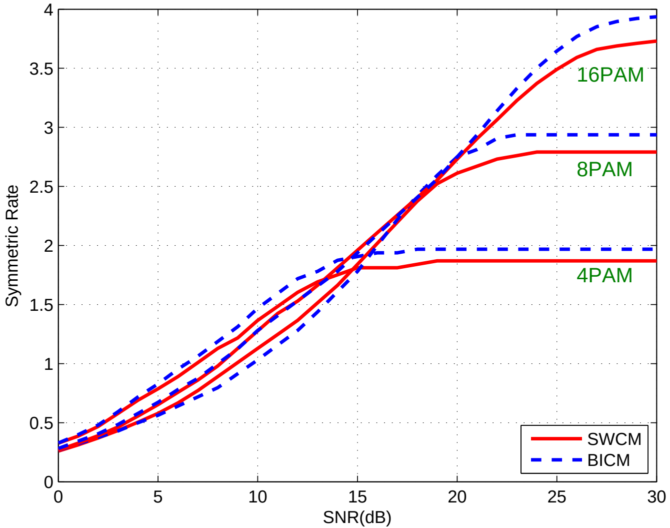

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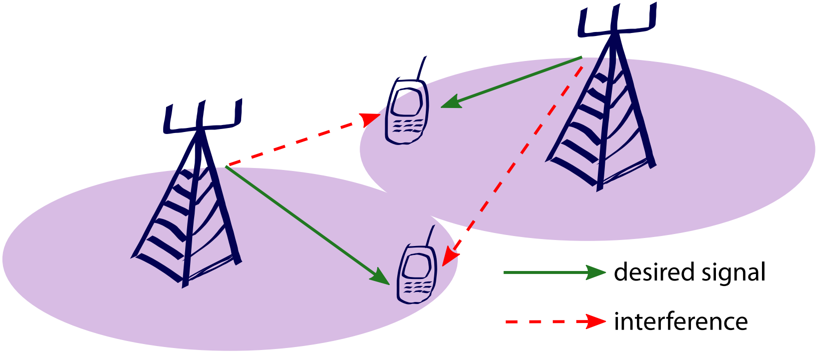
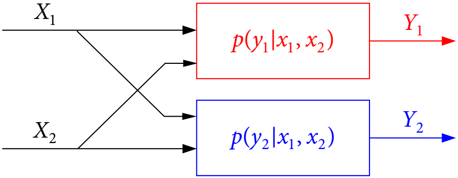
Application: Interference channels



Optimal rate region (Bandemer–El-Gamal–Kim 2012)



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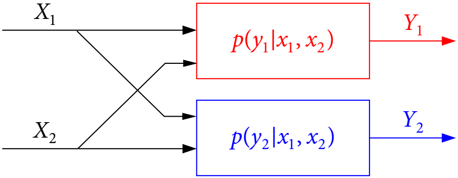

$$R_1 < I(X_1; Y_1|X_2)$$
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or

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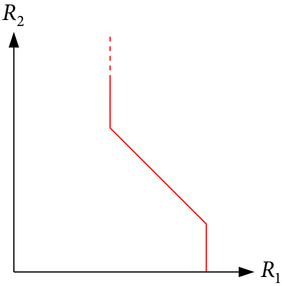

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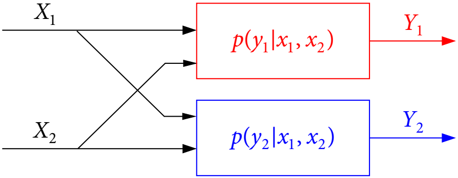

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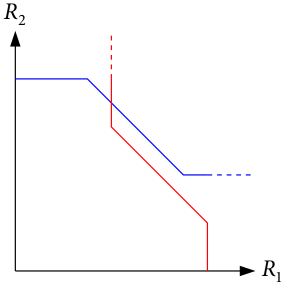
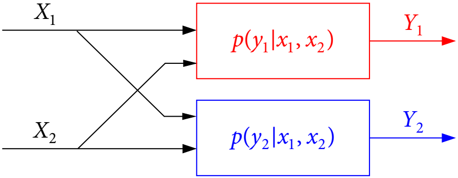


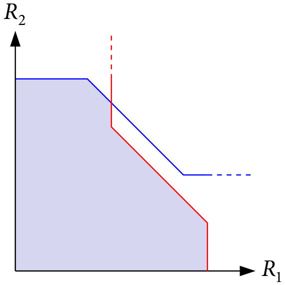
Optimal rate region (Bandemer–El-Gamal–Kim 2012)



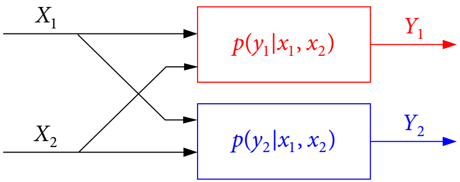
$$R_1 < I(X_1; Y_1|X_2)$$
$$R_1 + R_2 < I(X_1, X_2; Y_1)$$

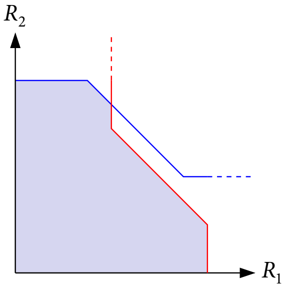
or

$$R_1 < I(X_1; Y_1)$$

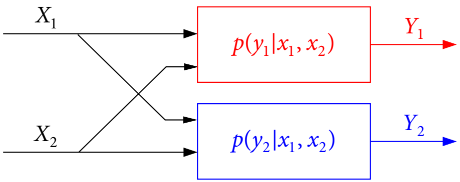


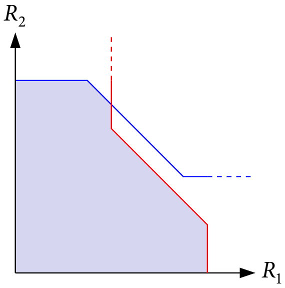
Low-complexity (implementable) alternatives





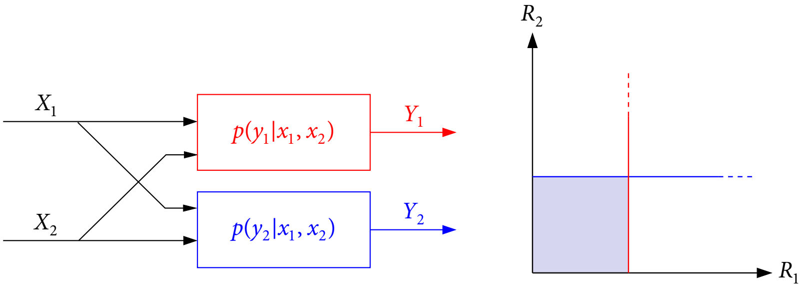
Low-complexity (implementable) alternatives





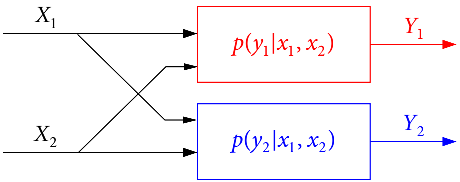
- P2P decoding

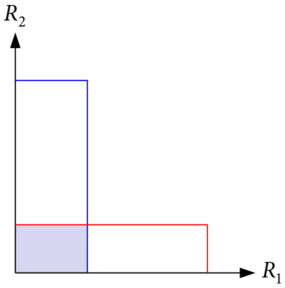
Low-complexity (implementable) alternatives



- P2P decoding
  - Treating interference as (Gaussian) noise: $R_1 < I(X_1; Y_1)$

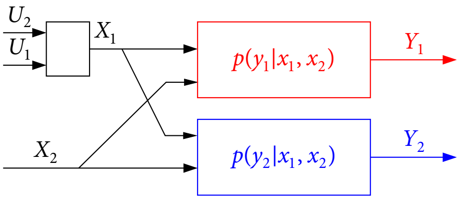
Low-complexity (implementable) alternatives

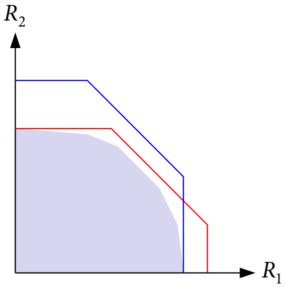




- P2P decoding
  - Treating interference as (Gaussian) noise: $R_1 < I(X_1; Y_1)$
  - Successive cancellation decoding: $R_2 < I(X_2; Y_1)$, $R_1 < I(X_1; Y_1|X_2)$

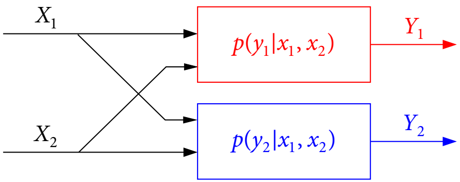
Low-complexity (implementable) alternatives

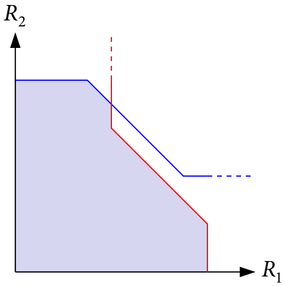




- P2P decoding
  - Treating interference as (Gaussian) noise: $R_1 < I(X_1; Y_1)$
  - Successive cancellation decoding: $R_2 < I(X_2; Y_1)$, $R_1 < I(X_1; Y_1|X_2)$
- + rate splitting (Zhao et al. 2011, Wang et al. 2014)

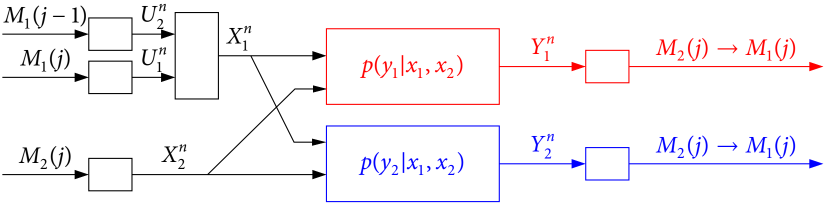
Low-complexity (implementable) alternatives



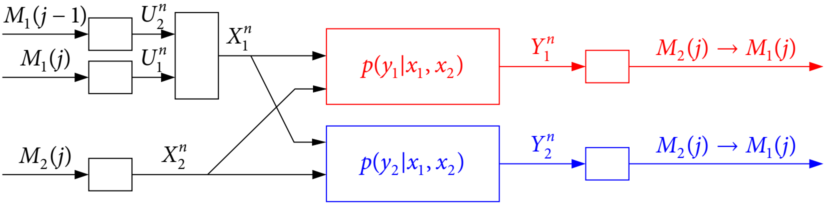


- P2P decoding
  - Treating interference as (Gaussian) noise: $R_1 < I(X_1; Y_1)$
  - Successive cancellation decoding: $R_2 < I(X_2; Y_1),\ R_1 < I(X_1; Y_1|X_2)$
- + rate splitting (Zhao et al. 2011, Wang et al. 2014)
- Novel codes
  - Spatially coupled codes (Yedla, Nguyen, Pfister, and Narayanan 2011)
  - Polar codes (Wang and Şaşoğlu 2014)

Sliding-window superposition coding (Wang et al. 2014)

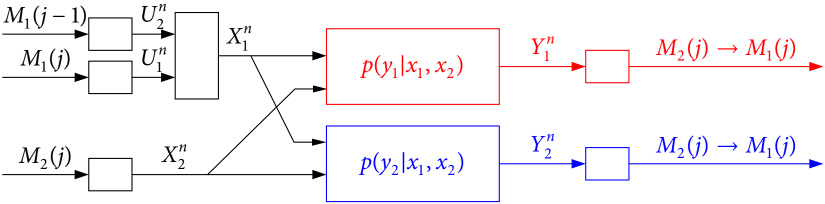


Sliding-window superposition coding (Wang et al. 2014)



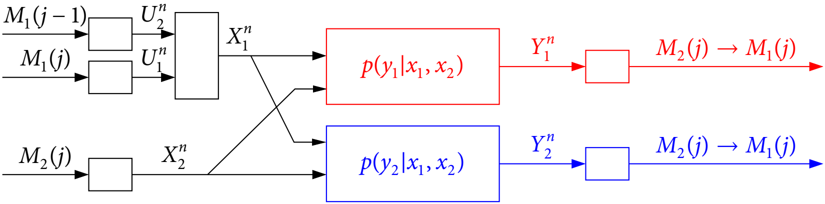
Block	1	2	3	4	5	6	7
U_2							
U_1							
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

Sliding-window superposition coding (Wang et al. 2014)



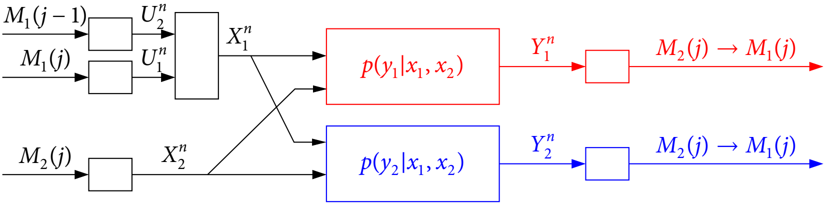
Block	1	2	3	4	5	6	7
U_2		$M_1(1)$					
U_1	$M_1(1)$						
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

Sliding-window superposition coding (Wang et al. 2014)



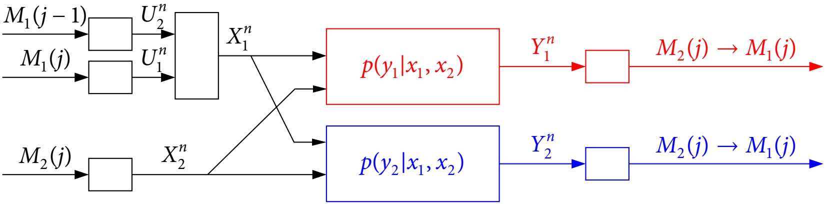
Block	1	2	3	4	5	6	7
U_2		$M_1(1)$	$M_1(2)$				
U_1	$M_1(1)$	$M_1(2)$					
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

Sliding-window superposition coding (Wang et al. 2014)



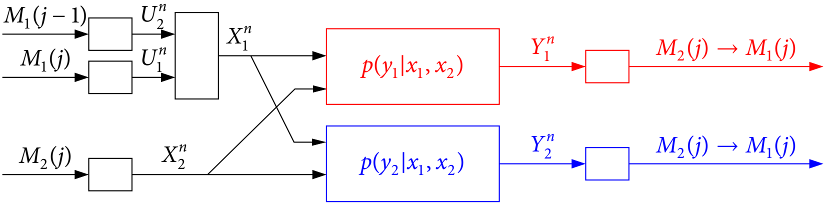
Block	1	2	3	4	5	6	7
U_2		$M_1(1)$	$M_1(2)$	$M_1(3)$			
U_1	$M_1(1)$	$M_1(2)$	$M_1(3)$				
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

Sliding-window superposition coding (Wang et al. 2014)



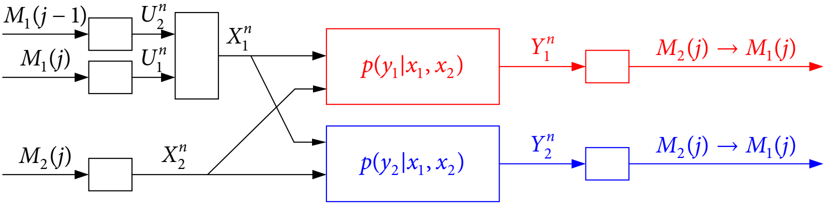
Block	1	2	3	4	5	6	7
U_2		$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$		
U_1	$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$			
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

Sliding-window superposition coding (Wang et al. 2014)



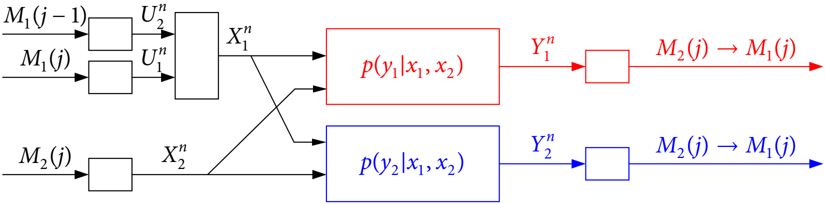
Block	1	2	3	4	5	6	7
U_2		$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	
U_1	$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$		
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

Sliding-window superposition coding (Wang et al. 2014)



Block	1	2	3	4	5	6	7
U_2		$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$
U_1	$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$	
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

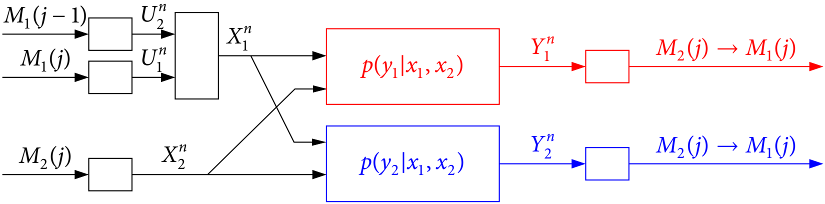
Sliding-window superposition coding (Wang et al. 2014)



Block	1	2	3	4	5	6	7
U_2		$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$
U_1	$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$	
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

- Sliding-window coded modulation for sender 1 (without alphabet constraints)

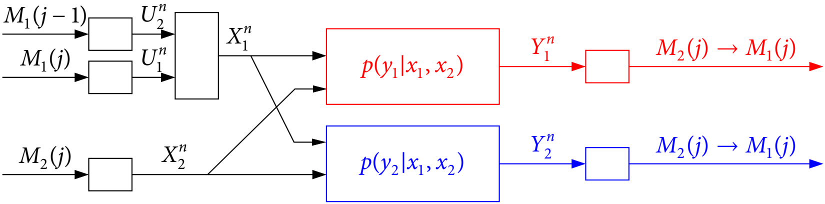
Sliding-window superposition coding (Wang et al. 2014)



Block	1	2	3	4	5	6	7
U_2		$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$
U_1	$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$	
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

- Sliding-window decoding

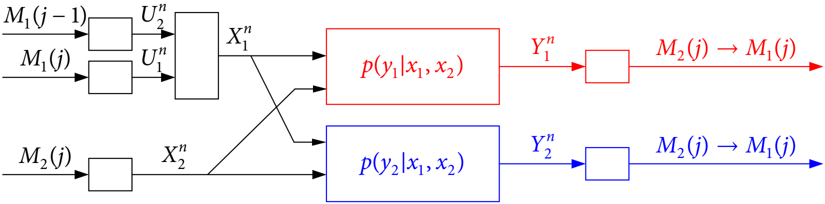
Sliding-window superposition coding (Wang et al. 2014)



Block	1	2	3	4	5	6	7
U_2			$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$
U_1		$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$	
X_2		$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

- Sliding-window decoding
- Successive cancellation decoding

Sliding-window superposition coding (Wang et al. 2014)

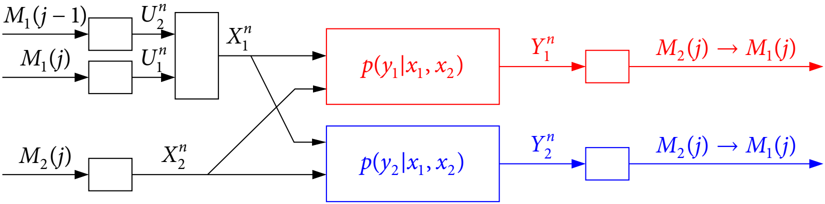


Block	1	2	3	4	5	6	7
U_2				$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$
U_1			$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$	
X_2			$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

- Sliding-window decoding
- Successive cancellation decoding

$$R_2 < I(X_2; Y_j | U_2)$$

Sliding-window superposition coding (Wang et al. 2014)



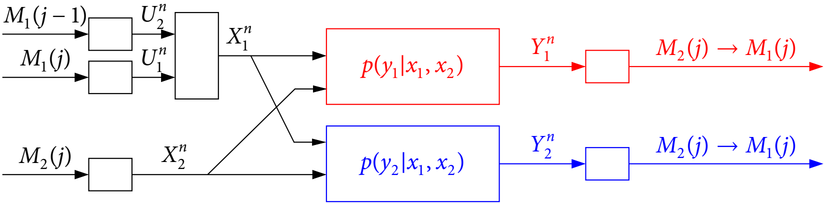
Block	1	2	3	4	5	6	7
U_2				$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$
U_1			$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$	
X_2				$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

- Sliding-window decoding
- Successive cancellation decoding

$$R_2 < I(X_2; Y_j | U_2)$$

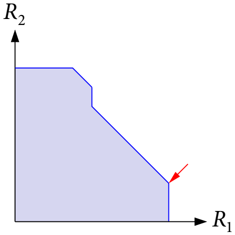
$$R_1 < I(U_2; Y_j) + I(U_1; Y_j | U_2, X_2)$$

Sliding-window superposition coding (Wang et al. 2014)

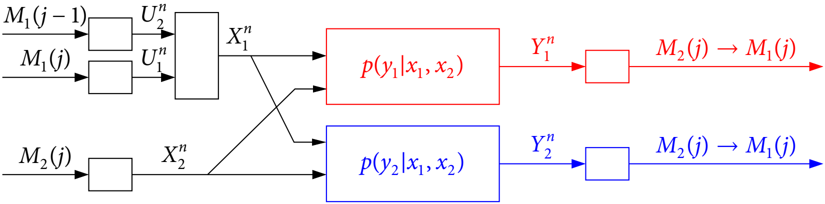


Block	1	2	3	4	5	6	7
U_2		$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$
U_1	$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$	
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

- Every corner point: different decoding orders

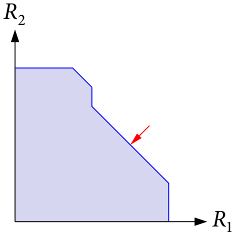


Sliding-window superposition coding (Wang et al. 2014)

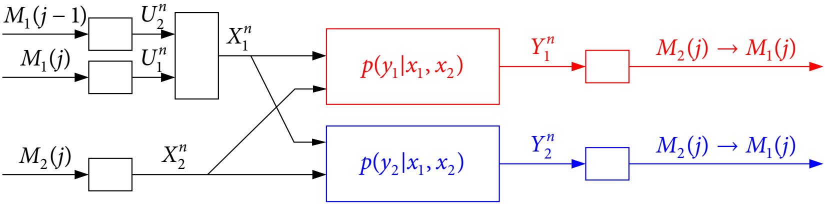


Block	1	2	3	4	5	6	7
U_2		$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$
U_1	$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$	
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

- Every corner point: different decoding orders
- Every point: time sharing or more superposition layers

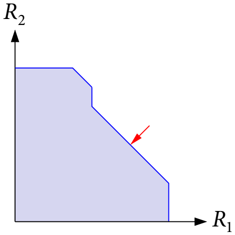


Sliding-window superposition coding (Wang et al. 2014)

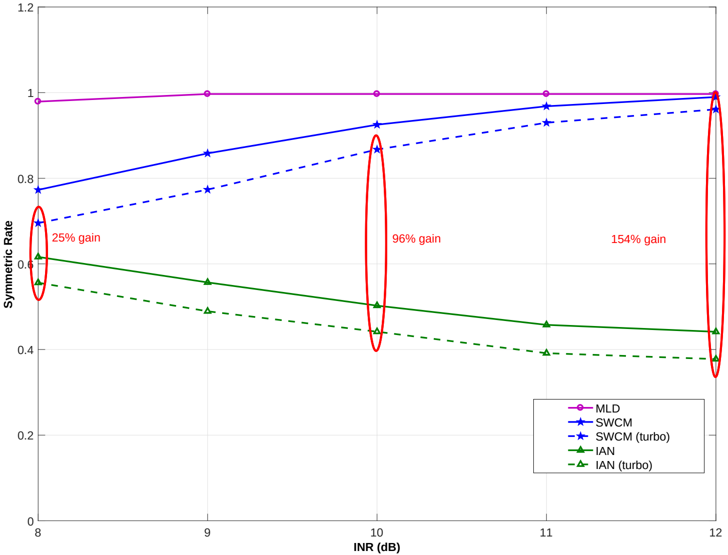


Block	1	2	3	4	5	6	7
U_2		$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$
U_1	$M_1(1)$	$M_1(2)$	$M_1(3)$	$M_1(4)$	$M_1(5)$	$M_1(6)$	
X_2	$M_2(1)$	$M_2(2)$	$M_2(3)$	$M_2(4)$	$M_2(5)$	$M_2(6)$	$M_2(7)$

- Every corner point: different decoding orders
- Every point: time sharing or more superposition layers
- Extension to Han–Kobayashi (Wang et al. 2017)

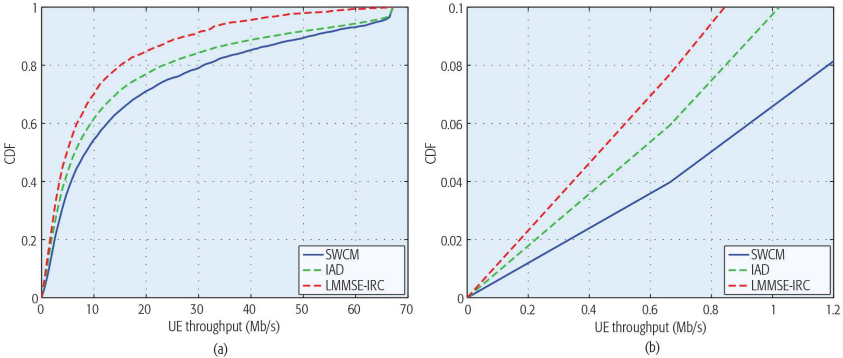


Gaussian channel performance (Park–Kim–Wang 2014)



LTE turbo code with $b = 20$, $n = 2048$, BLER $= 0.1$, SNR $= 10$ dB

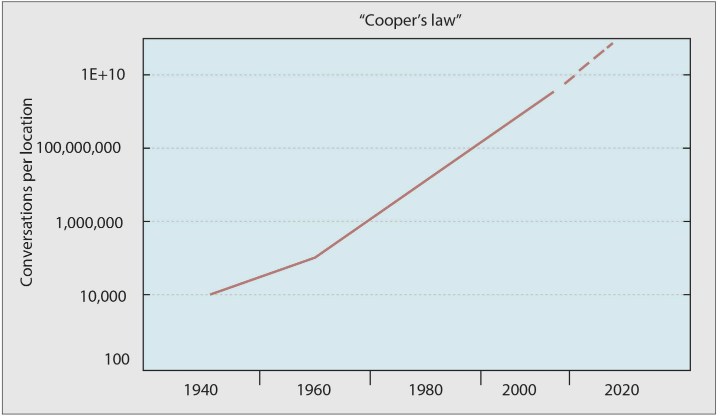
System-level performance (Kim et al. 2016)



Areal throughput (Mb/s/km^2)	Average UE throughput (Mb/s) (gain over baseline)			5% UE throughput (Mb/s) (gain over baseline)		
	LMMSE-IRC (baseline)	IAD	SWCM	LMMSE-IRC (baseline)	IAD	SWCM
33.6	16.921	21.122 (24.8%)	23.464 (38.7%)	0.981	1.189 (21.2%)	1.425 (45.3%)
57.22	10.996	14.252 (29.6%)	17.086 (55.4%)	0.471	0.583 (23.7%)	0.808 (71.5%)

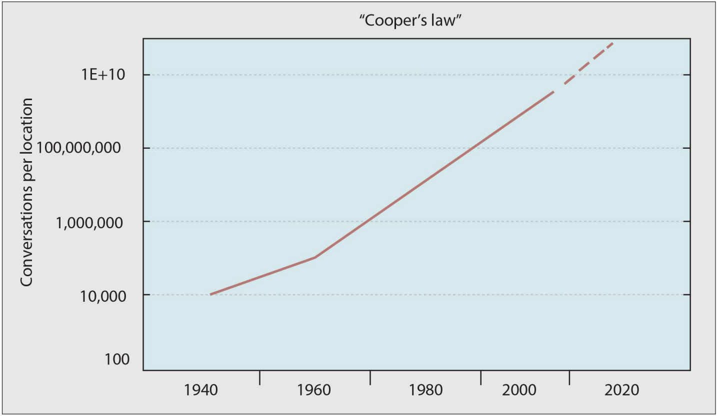
(c)

Cooper’s Law



Source: Arraycomm, Zander–Mähönen (2013)

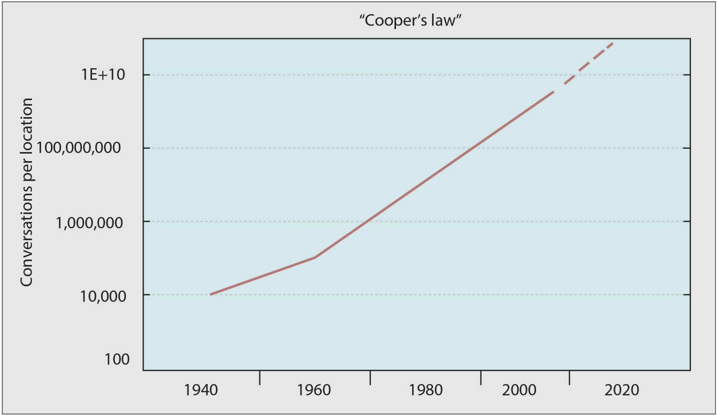
Cooper's Law



Source: Arraycomm, Zander–Mähönen (2013)

- Gain over the past 45 years $= 10^6 \propto \eta W_{\mathrm{sys}} N_{\mathrm{BS}}$

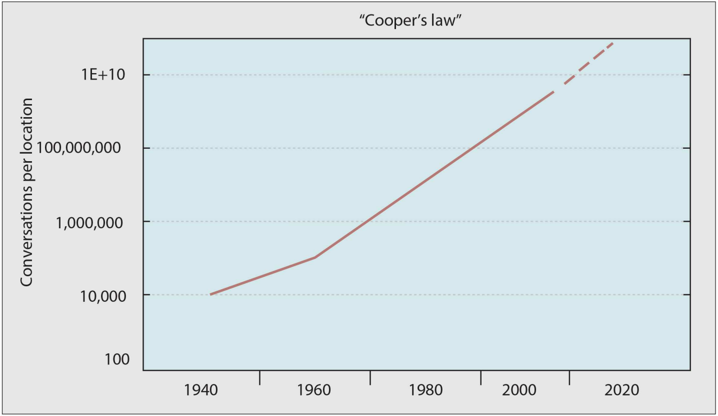
Cooper's Law



Source: Arraycomm, Zander–Mähönen (2013)

- Gain over the past 45 years = $10^6 \propto \eta W_{\text{sys}} N_{\text{BS}}$
  - Spectral efficiency η: x 25

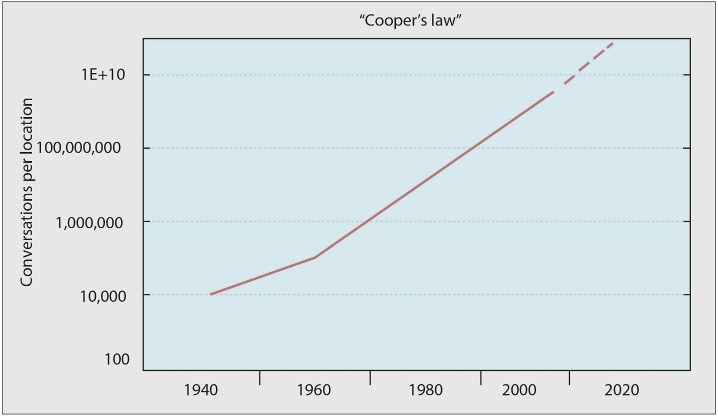
Cooper's Law



Source: Arraycomm, Zander–Mähönen (2013)

- Gain over the past 45 years = $10^6 \propto \eta W_{\text{sys}} N_{\text{BS}}$
  - Spectral efficiency η: x 25
  - System bandwidth W_{sys}: x 25

Cooper's Law



Source: Arraycomm, Zander–Mähönen (2013)

- Gain over the past 45 years = $10^6 \propto \eta W_{\mathrm{sys}} N_{\mathrm{BS}}$
  - Spectral efficiency η: x 25
  - System bandwidth W_{sys}: x 25
  - # of base stations N_{BS}: x 1600 (spatial reuse of frequency)

Concluding remarks

- Coded modulation as superposition coding

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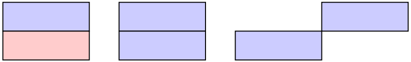
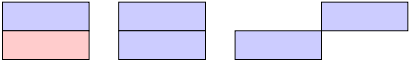
- Coded modulation as superposition coding
  - Simple and unifying picture

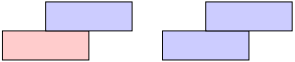


Concluding remarks

- Coded modulation as superposition coding
  - Simple and unifying picture



  - Framework for new coded modulation schemes



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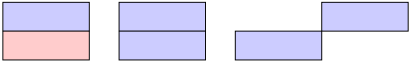

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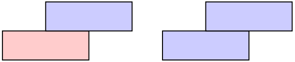

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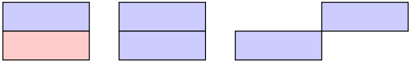

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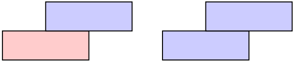

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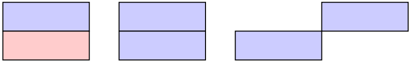

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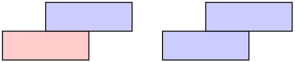

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- To learn more
  - Kramer and Kim (2018), "Network information theory for cellular wireless," in *Information Theoretic Perspectives on 5G Systems and Beyond,* eds. Shamai, Simeone, and Maric
  - Wang et al. (2017), "Sliding-window superposition coding: Two-user interference channels," arXiv:1701.02345
  - Kim et al. (2016), "Interference management via sliding-window coded modulation for 5G cellular networks," *IEEE Commun. Mag.*