Multiple User Writing on Dirty Paper

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I. Introduction

In "writing on dirty paper (WDP)" [1], Costa considered a variation of the standard additive white Gaussian noise (AWGN) channel with the channel output given by $Y^n = X^n + Z^n + S^n$. Here the n-block input X^n has average power constraint, the noise Z^n is white Gaussian, and the state noise S^n is white Gaussian and independent of Z^n. Costa showed that the capacity when the state information S^n is *noncausally* available at the transmitter, i.e., the transmitter knows the state sequence S^n ahead of the actual transmission, is equal to the capacity when S^n is available at both transmitter and receiver, which is, in turn, equal to the capacity of the standard AWGN channel without the additive state noise. In other words, the Gaussian noise S^n noncausally known at the transmitter does not affect the capacity of the AWGN channel. We call this the *WDP property.*

We establish the WDP property for three Gaussian multiple user channels with known capacity region — the Gaussian broadcast channel, the Gaussian multiple access channel, and the physically degraded Gaussian relay channel. Following [2–4], we can extend these results to 1) any stationary ergodic (possibly non-Gaussian) state and stationary Gaussian noise distribution, 2) any colored (possibly non-stationary) Gaussian state and noise distribution, and 3) any arbitrary individual state sequence and any colored (possibly non-stationary) Gaussian noise distribution, provided that the transmitter and the receiver share common randomness.

II. Main Results

We first consider a Gaussian broadcast channel with additive state

$$Y_1^n = X^n + S^n + Z_1^n, \tag{1a}$$
$$Y_2^n = X^n + S^n + Z_2^n, \tag{1b}$$

where the channel input X^n has an average power constraint, the arbitrarily correlated (not necessarily jointly Gaussian) noises Z_1^n and Z_2^n are respectively white Gaussian, and the state S^n is independent of (Z_1^n, Z_2^n) and is white Gaussian.

Theorem 1 *For the Gaussian broadcast channel (1), the capacity region when the state information S^n is noncausally available at the transmitter is equal to the capacity region when S^n is available at the transmitter and both receivers, which is, in turn, equal to the capacity region of the standard Gaussian broadcast channel without additive state noise.*

Next we consider a Gaussian multiple access channel with additive state

$$Y^n = X_1^n + X_2^n + S^n + Z^n, \tag{2}$$

where the channel inputs X_1^n and X_2^n have average power constraints, the noise Z^n is white Gaussian, and the state S^n is white Gaussian and independent of Z^n.

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Theorem 2 *For the Gaussian multiple access channel (2), the capacity region when the state information S^n is noncausally available at both transmitters is equal to the capacity region when S^n is available at the transmitters and the receiver, which is, in turn, equal to the capacity region of the standard Gaussian multiple access channel without additive state noise.*

Finally we establish the WDP property for the physically degraded relay channel with the relay and the receiver outputs respectively given by

$$Y_1^n = X^n + S^n + Z_1^n, \tag{3a}$$
$$Y^n = Y_1^n + X_1^n + Z_2^n = X^n + X_1^n + S^n + Z_1^n + Z_2^n. \tag{3b}$$

Here the channel input X^n and the relay input X_1^n have average power constraints, the noises Z_1^n, Z_2^n, and the state S^n are white Gaussian and independent of one another.

Theorem 3 *For the physically degraded Gaussian relay channel (3), the capacity when the state information S^n is noncausally available at both the transmitter and the relay is equal to the capacity when S^n is available at the transmitter, the relay, and the receiver, which is, in turn, equal to the capacity of the standard physically degraded Gaussian relay channel without additive state noise.*

The proofs use the achievable rate regions of corresponding discrete channels. Although those regions may be suboptimal in general, they turn out to be optimal for these Gaussian channels. Alternative proofs can be obtained by successive uses of Costa's WDP coding scheme, from which the WDP property can be established for more general noise and state distributions in a straightforward manner.

A natural question arises whether all Gaussian multiple user channels have the WDP property. The answer is no; in fact, one can show that the Gaussian strong interference channel does not have the WDP property.

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