Joint Source–Channel Coding via Hybrid Coding

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***Abstract*—A new analog-digital hybrid coding architecture for joint source–channel coding is proposed. The encoder generates a channel input by a symbol-by-symbol mapping of the observed (analog) source and its (digital) compression codeword, while the decoder reconstructs the source by a symbol-by-symbol mapping of the (analog) channel output and the decoded (digital) compression codeword from it. When applied to the problem of lossy communication of sources over the two-user discrete memoryless interference channel, this hybrid coding scheme achieves the best known performance and recovers as special cases several previous results on lossless and lossy communication over single hop networks.**

I. Introduction

Shannon's source-channel separation theorem states that a source can be optimally transmitted over a point-to-point discrete memoryless channel by concatenating a rate-distortion achieving source coder, which compresses the analog source into "bits" by removing the source redundancy, followed by a capacity achieving channel coder, which ensures that digital "bits" can be correctly decoded at the receiver by adding the transmission redundancy. The great appeal of this architecture is that the source coding and the channel coding operations can be performed independently of each other. It is well known, however, that this modular scheme yields suboptimal performance for the general problem of lossy transmission of correlated sources over multi-user discrete memoryless channels [1]. One of the great challenges towards developing a general theory of lossy communication over networks lies in designing new techniques for jointly designing source-channel codes.

In this paper we make one first step in this direction by proposing a new approach to joint source-channel coding based on hybrid coding, whereby the encoder generates a channel input by a symbol-by-symbol mapping of the observed (analog) source and its (digital) compression codeword, while the decoder reconstructs the source by a symbol-by-symbol mapping of the (analog) channel output and the decoded (digital) compression codeword from it. Shannon's source-channel separation scheme is recovered in the special case where the channel input is independent of the source and only carries information about the (digital) compression codeword. On the other hand, if the channel input is a function of the source only, we recover the uncoded (analog) transmission strategy.

Hybrid coding has been previously proposed as an alternative to Shannon's source-channel separation architecture [2], [3], [4], [5]. However, while most existing works in the literature focus on communication of Gaussian sources over additive Gaussian noise channels under squared error distortion measure, we propose a new generalized architecture that can be applied to any lossy communication problem over discrete memoryless channels. This new architecture is conceptually as simple as the traditional architecture of separate source and channel coding, yet it achieves much improved performance. To illustrate this approach, we study the problem of lossy communication of correlated sources over the two-user discrete memoryless interference channel in which each receiver estimates a *function* of the sources. For this problem, hybrid coding achieves the best known performance and recovers as special cases several previous results on lossless and lossy communication over single hop networks. The main contribution of the paper, however, lies not with the generality of the new scheme that unifies these results, but with the simple joint source–channel coding system architecture that is used in the proof of achievability and that could be useful for other multi-user communication problems.

The new joint source–channel coding system architecture is described first in the simple point-to-point communication setting in Section II and then applied to the problem of lossy communication over the the interference channel in Section III.

II. Background on Hybrid Coding

Consider the general point-to-point communication system depicted in Fig. 1, where the source $S \sim p(s)$ is to be communicated over the discrete memoryless channel $p(y|x)$. What is the sufficient and necessary condition such that the

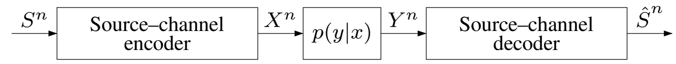


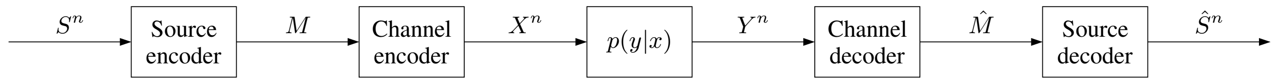
Fig. 1. Point-to-point communication system.

source can be reconstructed with expected distortion satisfying

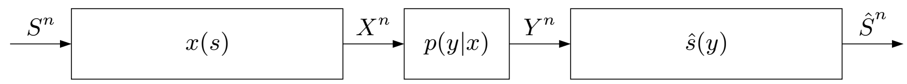
$$\frac{1}{n}\sum_{i=1}^{n} \mathsf{E}(d(S_i,\hat{S}_i)) \le D?$$

Shannon [6], [7] showed that a distortion D is achievable if

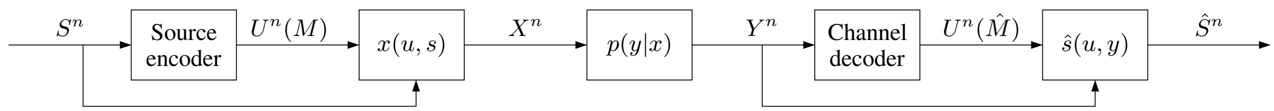
$$R(D) < C, \qquad (1)$$



(a) Separate source and channel coding system architecture.



(b) Uncoded transmission.



(c) A new joint source–channel coding system architecture based on hybrid coding.

Fig. 2. Three system architectures for the problem of lossy transmission of a source over a point to-point channel.

where $R(D) = \min_{p(\hat{s}|s):\, \mathsf{E}(d(S,\hat{S}))\le D} I(S;\hat{S})$ is the rate–distortion function for the source S and distortion measure $d(s,\hat{s})$ and

$$C = \max_{p(x)} I(X;Y)$$

is the capacity of the channel $p(y|x)$. The proof of this result uses separate source and channel coding, as illustrated in Fig. 2(a). Under this separate source and channel coding architecture, the source sequence is mapped into one of 2^{nR} indices M and then this index is mapped into a channel codeword X^n, which is transmitted over the channel. Upon receiving Y^n, the decoder finds an estimate $\hat{M}$ of M and reconstructs $\hat{S}^n$ from $\hat{M}$. The index M provides a digital interface between the source code and the channel code, which can be designed and operated separately. By the lossy source coding theorem and the channel coding theorem, the desired distortion D can be achieved, provided that the index rate R satisfies $R > R(D)$ and $R < C$.

As another extreme point of communication, we can consider uncoded transmission as depicted in Fig. 2(b). In this simple analog interface between the source and the channel, the encoder transmits a symbol-by-symbol mapping of the source sequences. Despite its simplicity, this uncoded transmission can be sometimes optimal, for example, when communicating a Gaussian source over a Gaussian channel under a squared error (quadratic) distortion measure [8] or communicating a binary source over a binary symmetric channel under a Hamming distortion measure. In these two cases, the desired distortion D can be achieved if $C \ge R(D)$. (Note the nonstrict inequality, compared to Shannon's sufficient condition for source–channel separation.)

Hybrid coding combines the separate source and channel coding and uncoded transmission (see Fig. 2(c)). Under this new architecture, the source sequence S^n is mapped to one of 2^{nR} sequences $U^n(M)$ and then this sequence $U^n(M)$ (along with S^n) is mapped to X^n *symbol-by-symbol*, which is transmitted over the channel. Upon receiving Y^n, the decoder finds an estimate $U^n(\hat{M})$ of $U^n(M)$ and reconstructs $\hat{S}^n$ from U^n (and Y^n) again by a *symbol-by-symbol* mapping. Thus, the codeword $U^n(M)$ plays the roles of both the source codeword $\hat{S}^n(M)$ and the channel codeword $X^n(M)$ simultaneously. This dual role of $U^n(M)$ allows simple symbol-by-symbol interfaces $x(u,s)$ and $\hat{s}(u,y)$ that replace the channel encoder and the source decoder in the separation architecture. Moreover, the source encoder and the channel decoder can be operated separately. Roughly speaking, again by the lossy source coding theorem, the condition $R > I(U;S)$ guarantees a reliable source encoding operation and by the channel coding theorem, the condition $R < I(U;Y)$ guarantees a reliable channel decoding operation (over the channel $p(y|u) = \sum_s p(y|x(u,s))p(s)$). Thus, a distortion D is achievable if

$$I(S;U) < I(U;Y) \tag{2}$$

for some pmf $p(u|s)$ and functions $x(u,s)$ and $\hat{s}(u,y)$ such that $\mathsf{E}(d(S,\hat{S})) \le D$. By taking $U = (X,\hat{S})$ where $\hat{S} \sim p(\hat{s}|s)$, $X \sim p(x)$ is independent of S and $\hat{S}$, and using the memoryless property of the channel, it can be easily shown that this condition simplifies to (1). On the other hand, by taking $U = \emptyset$, hybrid coding reduces to uncoded transmission.

Conceptually speaking, this new coding scheme is as simple as the separation scheme. The precise analysis of its performance, however, involves a technical subtlety. In particular, because $U^n(M)$ is used as a source codeword, the index M depends on the entire codebook $\mathcal{C} = \{U^n(M) : M \in [1 : 2^{nR}]\}$. But the conventional random coding proof technique for a channel codeword $U^n(M)$ is developed for situations for which the index M and the (random) codebook $\mathcal{C}$ are independent of each other. The dependency issue for joint source–channel coding has been well noted by Lapidoth and Tinguely [4, Proof of Proposition D.1], who developed a geometric approach for sending a bi-variate Gaussian source over a Gaussian multiple access channel. For a detailed proof of the achievability of hybrid coding with nonstandard analysis that overcomes this difficulty, refer to [9].

III. Joint source-channel coding over the interference channel

The problem of lossy communication of a pair of correlated discrete memoryless sources $(S_1, S_2) \sim p(S_1, S_2)$ over the discrete memoryless interference channel $p(y_1, y_2|x_1, x_2)$ depicted in Fig. 3 is the most general communication problem

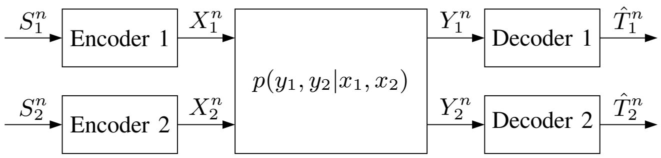


Fig. 3. Communication of a 2-DMS over a DM-IC.

for single-hop networks, i.e., networks where each nodes can be either a source or a destination but not both, and in fact it includes as special cases the problems of lossy communication over the two-user multiple access and broadcast channels. Here each sender $j = 1, 2$ wishes to communicate its source S_j over the channel so that receiver j can compute a function T_j of the sources with desired distortion. We assume, for simplicity, that the sources S_1 and S_2 have no common part in the sense of Gács–Körner [10] and Witsenhausen [11] and consider the block coding setting in which the source sequences $S_1^n = (S_{11}, \ldots, S_{1n})$ and $S_2^n = (S_{21}, \ldots, S_{2n})$ are communicated by n transmissions over the channel. Formally, a $(|\mathcal{S}_1|^n, |\mathcal{S}_2|^n, n)$ joint source–channel code for this problem consists of

- two encoders, where encoder $j = 1, 2$ assigns a sequence $x_j^n(s_j^n)$ to each source sequence s_j^n, and
- two decoders, where decoder $j = 1, 2$ assigns an estimate $\hat{t}_j^n$ to each channel output sequence y_j^n.

Let $d_1(s_1, s_2, \hat{t}_1)$ and $d_2(s_1, s_2, \hat{t}_2)$ be two distortions measures. The average per-letter distortion $d_j(s_1^n, s_2^n, \hat{t}_j^n)$, $j = 1, 2$, is defined as $d_j(s_1^n, s_2^n, \hat{t}_j^n) = (1/n)\sum_{i=1}^n d_j(s_{1i}, s_{2i}, \hat{t}_{ji})$. A distortion pair (D_1, D_2) is said to be achievable for communication of the sources (S_1, S_2) over the interference channel $p(y_1, y_2|x_1, x_2)$ if there exists a sequence of $(|\mathcal{S}_1|^n, |\mathcal{S}_2|^n, n)$ joint source–channel codes such that

$$\limsup_{n\to\infty} \mathsf{E}(d_j(S_1^n, S_2^n, \hat{T}_j^n)) \le D_j, \quad j = 1, 2.$$

The optimal distortion region is the closure of the set of all achievable distortion pairs (D_1, D_2). A computable characterization of the optimal distortion region for this problem is not known. Even the simpler question of characterizing the capacity region when sending independent messages over this channel is a long-standing open problem.

The hybrid coding architecture outlined above can be used to characterize a computable inner bound on the optimal distortion region. At a high level, the proposed approach consists in concatenating the lossy version of the coding scheme of Gray and Wyner [12] for source coding with a generalized version of the channel coding scheme of Han and Kobayashi for the interference channel [13]. Specifically, consider the hybrid coding architecture depicted in Fig. 4, wherein each source sequence S_j^n, $j = 1, 2$, is mapped to one of 2^{nR_j} sequences $W_j(m_j)$, then each pair $(S_j^n, W_j^n(m_j))$ is mapped to one of $2^{nR_{1j}+R_{2j}}$ sequence pairs $(U_j^n(m_j, l_j), V_j^n(m_j, k_j))$. As in the source coding scheme scheme of Gray and Wyner, $W_j^n(m_j)$ encodes common (digital) information decoded by both receivers, while U^n and V^n serve as satellite codewords used to transmit additional private information. Encoding is performed using joint typicality encoding and standard information theoretic techniques can be used to determine the conditions for successful encoding. Then, the source and the codewords corresponding to the three (digital) messages (m_j, l_j, k_j) are mapped symbol-by-symbol to the sequence X_j^n that is transmitted over the interference channel. Upon receiving Y_j^n, decoder j recovers

$$W_j^n(m_j), U^n(m_j, k_j), W_{j^c}^n(m_{j^c}), U^n(m_{j^c}, k_{j^c})$$

by joint typicality decoding. Here and throughout, we use the notation $j^c = \{1, 2\} \setminus \{j\}$. Notice that decoder j decodes part of the information transmitted by encoder j^c as in the channel coding theorem of Han and Kobayashi for the interference channel. Once we ignore the issue of the dependence between the indices and the codebook that we have mentioned in the discussion of the point-to-point channel, the conditions for successful decoding the covering indexes can be obtained by applying the packing lemma to the multiple access channel $p(y_j|w_j, u_j, w_{j^c}, v_{j^c})$. Then, decoder j reconstructs $\hat{T}_j$ by mapping symbol-by-symbol the analog channel output Y_j^n and the codewords corresponding to four decoded digital messages.

A. Main Result

Hybrid coding yields the following inner bound on the optimal distortion region.

Theorem 1: A distortion pair (D_1, D_2) is achievable for communication of the sources (S_1, S_2) over the interference channel $p(y_1, y_2|x_1, x_2)$ if there exist a pmf

$$p(q)p(w_1, u_1, v_1|s_1, q)p(w_2, u_2, v_2|s_2, q),$$

two encoding functions $x_j(w_j, u_j, v_j, s_j, q)$, and two decoding functions $\hat{t}_j(w_j, u_j, w_{j^c}, v_{j^c}, y_j, q)$ such that, for each $j = 1, 2$,

$$\mathsf{E}(d_j(S_1, S_2, \hat{T}_j)) \le D_j$$

and the inequalities in (3) on the top of the last page are satisfied for some rate tuple $(R_{0j}, R_{1j}, R_{2j} : j \in \{1, 2\})$.

Remark 1: The above sufficient condition generalized several previous results in the literature including the ones of Berger [14] and Tung [15] for distributed lossy source coding, Lapidoth and Tinguely [4] for lossy communication of a bivariate Gaussian source over a Gaussian multiple access channel, Han and Costa [16] for lossless communication of correlated sources over a broadcast channel, and the result of Tian, Diggavi, and Shamai [5] for lossy communication of a bivariate Gaussian source over a Gaussian broadcast channel, and of Han and Kobayashi [13] for communication of independent messages over the interference channel. Furthermore, the above theorem improves upon the previous result by Liu and Chen [17] on lossy communication over the interference channel.

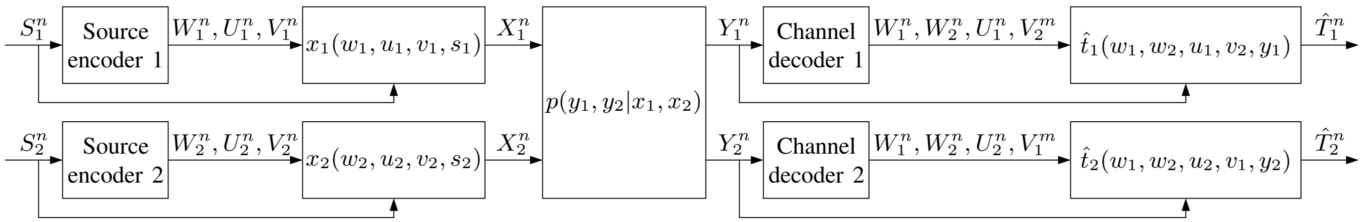


Fig. 4. System architecture for the problem of lossy communicating over an interference channel.

Remark 2: When specialized to the problem of lossless communication of correlated sources over multiple access channels, the sufficient condition in Theorem 1 reduces to the one by Cover, El Gamal, and Salehi [1]. Hence, Dueck's counterexample [18] shows that in general this sufficient condition is not necessary.

B. Outline of the achievability proof

In the following, we provide an outline for the achievability proof in the special case $Q = \emptyset$. Achievability for an arbitrary Q can be proved using coded time sharing technique [19].

Codebook generation: We randomly and independently generate a codebook for each $j = 1, 2$. Fix a pmf $p(w_j, u_j, v_j|s_j)$, an encoding function $x_j(w_j, u_j, v_j, s_j)$, and a reconstruction function $\hat{t}_j(w_j, u_j, w_{j^c}, v_{j^c}, y_j)$ such that the average distortion constraints are satisfied. Randomly and independently generate $2^{nR_{0j}}$ sequences $w_j^n(m_j)$, $m_j \in [1 : 2^{nR_{0j}}]$, each according to $\prod_{i=1}^n p_{W_j}(w_{ji})$. For each m_j, randomly and conditionally independently generate $2^{nR_{1j}}$ sequences $u_j^n(m_j, l_j)$, $l_j \in [1 : 2^{nR_{1j}}]$, each according to $\prod_{i=1}^n p_{U_j|W_j}(u_{ji}|w_{ji}(m_j))$. For each m_j, randomly and conditionally independently generate $2^{nR_{2j}}$ sequences $v_j^n(m_j, k_j)$, $k_j \in [1 : 2^{nR_{2j}}]$, each according to $\prod_{i=1}^n p_{V_j|W_j}(v_{ji}|w_{ji}(m_j))$.

Encoding: Fix $\epsilon > \epsilon' > 0$. Upon observing s_j^n, encoder $j = 1, 2$ finds an index triple $(m_j, l_j, k_j) \in [1 : 2^{nR_{0j}}] \times [1 : 2^{nR_{1j}}] \times [1 : 2^{nR_{2j}}]$ such that

$$(w_j^n(m_j), u_j^n(m_j, l_j), v_j^n(m_j, k_j), s_j^n) \in \mathcal{T}_{\epsilon'}^{(n)}.$$

If there is more than one such index triple, choose one of them at random. Otherwise, declare an error and choose an index triple at random. Encoder j transmits $x_{ji} = x_j(w_{ji}(m_j), u_{ji}(m_j, l_j), v_{ji}(m_j, k_j), s_{ji})$ at time $i = 1, \cdots, n$.

By the mutual covering lemma [19], the source encoding is successful if

$$\begin{aligned} R_{0j} &> I(W_j; S_j) + \delta(\epsilon'), \\ R_{1j} &> I(U_j; S_j|W_j) + \delta(\epsilon'), \\ R_{2j} &> I(V_j; S_j|W_j) + \delta(\epsilon'), \\ R_{1j} + R_{2j} &> I(U_j; S_j|W_j) + I(V_j; S_j|W_j) \\ &\quad + I(U_j; V_j|W_j, S_j) + \delta(\epsilon') \end{aligned}$$

for $j = 1, 2$.

Decoding: Upon observing the sequence y_j^n, decoder $j = 1, 2$ finds the unique index tuple $(\hat{m}_1, \hat{m}_2, \hat{l}_j, \hat{k}_{j^c})$ such that

$$(w_j^n(\hat{m}_j), u_j^n(\hat{m}_j, \hat{l}_j), w_{j^c}^n(\hat{m}_{j^c}), v_{j^c}^n(\hat{m}_{j^c}, \hat{k}_{j^c}), y_j^n) \in \mathcal{T}_\epsilon^{(n)}.$$

Then, decoder j declares

$$\hat{t}_{ji} = \hat{t}_j(w_{ji}(\hat{m}_j), u_{ji}(\hat{m}_j, \hat{l}_j), w_{j^c i}(\hat{m}_{j^c}), v_{j^c i}(\hat{m}_{j^c}, \hat{k}_{j^c}), y_{ji}),$$

$i \in [1 : n]$ as its estimate of t_{ji}, $i = 1, \cdots, n$.

In the following we outline the probability of error analysis for decoder 1. The analysis for decoder 2 follows similarly. Assume that the source encoding is successful and let (M_1, L_1, K_1) and (M_2, L_2, K_2) be the random variables denoting the chosen indices at the encoders 1 and 2, respectively. Then, the error event for decoder 1 can be partitioned into two events,

$$\begin{aligned} \mathcal{E}_1 = \{&(S_1^n, S_2^n, W_1^n(M_1), U_1^n(M_1, L_1), \\ &W_2^n(M_2), V_2^n(M_2, K_2), Y_1^n) \notin \mathcal{T}_\epsilon^{(n)}\}, \\ \mathcal{E}_2 = \{&(S_1^n, S_2^n, W_1^n(m_1), U_1^n(m_1, l_1), \\ &W_2^n(m_2), V_2^n(m_2, k_2), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \\ &\text{for some } (m_1, m_2, l_1, k_2) \neq (M_1, M_2, L_1, K_2)\}. \end{aligned}$$

By the Markov lemma [19, Lecture Note 13], $\mathsf{P}(\mathcal{E}_1)$ tends to zero as $n \to \infty$. To bound $\mathsf{P}(\mathcal{E}_2)$, let

$$\begin{aligned} \tilde{\mathcal{E}}(m_1, l_1, m_2, k_2) = \{&(S_1^n, S_2^n, W_1^n(m_1), U_1^n(m_1, l_1), \\ &W_2^n(m_2), V_2^n(m_2, k_2), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}\}. \end{aligned}$$

Then, the event $\mathcal{E}_2$ can be divided into the events that $\tilde{\mathcal{E}}(m_1, l_1, m_2, k_2)$ occurs for the following cases.

1) $\hat{m}_1 \neq M_1$, $\hat{l}_1 \neq L_1$, $\hat{m}_2 \neq M_2$, and $\hat{k}_2 \neq K_2$: The probability of this event tends to zero as $n \to \infty$ if

$$\begin{aligned} R_{01} + R_{11} + R_{02} + R_{22} < I(&W_1, U_1, W_2, V_2; Y_1) \\ &+ I(W_1, U_1; W_2, V_2) - \delta(\epsilon). \end{aligned}$$

2) $\hat{m}_1 = M_1$, $\hat{l}_1 \neq L_1$, $\hat{m}_2 \neq M_2$, and $\hat{k}_2 \neq K_2$: The probability of this event tends to zero as $n \to \infty$ if

$$\begin{aligned} R_{11} + R_{02} + R_{22} < I(&U_1, W_2, V_2; Y_1|W_1) \\ &+ I(W_1, U_1; W_2, V_2) - \delta(\epsilon). \end{aligned}$$

3) $\hat{m}_1 = M_1$, $\hat{l}_1 = L_1$, $\hat{m}_2 \neq M_2$, and $\hat{k}_2 \neq K_2$: The probability of this event tends to zero as $n \to \infty$ if

$$R_{02} + R_{22} < I(W_2, V_2; Y_1, W_1, U_1) - \delta(\epsilon).$$

$$
\begin{aligned}
R_{0j} &> I(W_j;S_j|Q),\\
R_{1j} &> I(U_j;S_j|W_j,Q),\\
R_{2j} &> I(V_j;S_j|W_j,Q),\\
R_{1j}+R_{2j} &> I(U_j;S_j|W_j,Q)+I(V_j;S_j|W_j,Q)+I(U_j;V_j|W_j,S_j,Q),\\
R_{1j} &< I(U_j;Y_j,W_{j^c},V_{j^c}|W_j,Q),\\
R_{2j^c} &< I(V_{j^c};Y_j,W_j,U_j|W_{j^c},Q),\\
R_{0j}+R_{1j} &< I(W_j,U_j;Y_j,W_{j^c},V_{j^c}|Q),\\
R_{0j^c}+R_{2j^c} &< I(W_{j^c},V_{j^c};Y_j,W_j,U_j|Q),\\
R_{1j}+R_{2j^c} &< I(U_j,V_{j^c};Y_j|W_j,W_{j^c},Q)+I(W_j,U_j;W_{j^c},V_{j^c}|Q)-I(W_j;W_{j^c}|Q),\\
R_{1j}+R_{0j^c}+R_{2j^c} &< I(U_j,W_{j^c},V_{j^c};Y_j|W_j,Q)+I(W_j,U_j;W_{j^c},V_{j^c}|Q),\\
R_{0j}+R_{1j}+R_{2j^c} &< I(W_j,U_j,V_{j^c};Y_j|W_{j^c},Q)+I(W_j,U_j;W_{j^c},V_{j^c}|Q),\\
R_{0j}+R_{1j}+R_{0j^c}+R_{2j^c} &< I(W_j,U_j,W_{j^c},V_{j^c};Y_j|Q)+I(W_j,U_j;W_{j^c},V_{j^c}|Q).
\end{aligned}
\tag{3}
$$

4) $\hat{m}_1 \neq M_1$, $\hat{l}_1 \neq L_1$, $\hat{m}_2 = M_2$, and $\hat{k}_2 \neq K_2$: The probability of this event tends to zero as $n \to \infty$ if

$$
\begin{aligned}
R_{01}+R_{11}+R_{22} < I(&W_1,U_1,V_2;Y_1|W_2)\\
&+I(W_1,U_1;W_2,V_2)-\delta(\epsilon).
\end{aligned}
$$

5) $\hat{m}_1 = M_1$, $\hat{l}_1 \neq L_1$, $\hat{m}_2 = M_2$, and $\hat{k}_2 \neq K_2$: The probability of this event tends to zero as $n \to \infty$ if

$$
\begin{aligned}
R_{11}+R_{22} < I(&U_1,V_2;Y_1|W_1,W_2)\\
&+I(W_1,U_1;W_2,V_2)-I(W_1;W_2)-\delta(\epsilon).
\end{aligned}
$$

6) $\hat{m}_1 = M_1$, $\hat{l}_1 = L_1$, $\hat{m}_2 = M_2$, and $\hat{k}_2 \neq K_2$: The probability of this event tends to zero as $n \to \infty$ if

$$R_{22} < I(V_2;Y_1,W_1,U_1|W_2)-\delta(\epsilon).$$

7) $\hat{m}_1 = M_1$, $\hat{l}_1 \neq L_1$, $\hat{m}_2 = M_2$, and $\hat{k}_2 = K_2$: The probability of this event tends to zero as $n \to \infty$ if

$$R_{11} < I(U_1;Y_1,W_2,V_2|W_1)-\delta(\epsilon).$$

Therefore, $\mathsf{P}(\mathcal{E}_2)$ tends to zero as $n \to \infty$ if the inequalities for all cases are satisfied. Finally, by the law of total expectation and the typical average lemma the desired distortion is achieved.

References

[1] T. M. Cover, A. El Gamal, and M. Salehi, "Multiple access channels with arbitrarily correlated sources," *IEEE Trans. Inf. Theory*, vol. 26, no. 6, pp. 648–657, Nov. 1980.

[2] S. Shamai, S. Verdu, and R. Zamir, "Systematic lossy source/channel coding," *IEEE Trans. Inf. Theory*, vol. 44, pp. 564–579, 1998.

[3] M. Wilson, K. Narayanan, and G. Caire, "Joint source channel coding with side information using hybrid digital analog codes," *IEEE Trans. Inf. Theory*, vol. 56, no. 10, pp. 4922–4940, 2010.

[4] A. Lapidoth and S. Tinguely, "Sending a bivariate Gaussian over a Gaussian MAC," *IEEE Trans. Inf. Theory*, vol. 56, no. 6, pp. 2714–2752, 2010.

[5] C. Tian, S. Diggavi, and S. S., "The achievable distortion region of bivariate Gaussian source on gaussian broadcast channel," 2010. [Online]. Available: http://arxiv.org/abs/1006.0644

[6] C. E. Shannon, "A mathematical theory of communication," *Bell System Tech. J.*, vol. 27, pp. 379–423, 623–656, 1948.

[7] ——, "Coding theorems for a discrete source with a fidelity criterion," in *IRE Int. Conv. Rec., part 4*, 1959, vol. 7, pp. 142–163, reprinted with changes in *Information and Decision Processes,* R. E. Machol, Ed. New York: McGraw-Hill, 1960, pp. 93-126.

[8] T. J. Goblick, "Theoretical limitations on the transmission of data from analog sources," *IEEE Trans. Inf. Theory*, vol. 11, pp. 558–567, 1965.

[9] S. H. Lim, P. Minero, and Y.-H. Kim, "Lossy communication of correlated sources over multiple access channels," in *Proc. 48th Annual Allerton Conference on Communications, Control, and Computing*, Monticello, IL, Oct. 2010.

[10] P. Gács and J. Körner, "Common information is far less than mutual information," *Probl. Control Inf. Theory*, vol. 2, no. 2, pp. 149–162, 1973.

[11] H. S. Witsenhausen, "On sequences of pairs of dependent random variables," *SIAM J. Appl. Math.*, vol. 28, pp. 100–113, 1975.

[12] R. M. Gray and A. D. Wyner, "Source coding for a simple network," *Bell System Tech. J.*, vol. 53, pp. 1681–1721, 1974.

[13] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, 1981.

[14] T. Berger, "Multiterminal source coding," in *The Information Theory Approach to Communications*, G. Longo, Ed. New York: Springer-Verlag, 1978.

[15] S.-Y. Tung, "Multiterminal source coding," Ph.D. Thesis, Cornell University, Ithaca, NY, 1978.

[16] T. S. Han and M. M. H. Costa, "Broadcast channels with arbitrarily correlated sources," *IEEE Trans. Inf. Theory*, vol. 33, no. 5, pp. 641–650, Sep. 1987.

[17] W. Liu and B. Chen, "Communicating correlated sources over interference channels: The lossy case," in *Proc. IEEE International Symposium on Information Theory*, Austin, Texas, 2010, pp. 345 –349.

[18] G. Dueck, "A note on the multiple access channel with correlated sources," *IEEE Trans. Inf. Theory*, vol. 27, no. 2, pp. 232–235, 1981.

[19] A. El Gamal and Y.-H. Kim, "Lecture notes on network information theory," 2010. [Online]. Available: http://arxiv.org/abs/1001.3404