Local Time Sharing for Index Coding

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***Abstract*—A series of extensions of the index coding schemes based on time sharing by Birk and Kol, by Blasiak, Kleinberg, and Lubetzky, and by Shanmugam, Dimakis, and Langberg are presented. Each extension strictly improves upon the previous extensions as well as the existing schemes. The main idea behind these extensions is local time sharing over subproblems introduced by Shanmugam et al., in which the local side information available at each receiver is exploited to send the subproblm indices with a fewer number of transmissions. The final extension, despite being the best in this class of coding schemes, is shown to be still suboptimal, characterizing the fundamental limit of local time sharing.**

I. Introduction

Consider a communication scenario, referred to as *index coding*, in which a sender wishes to communicate a tuple of n messages, $x^n = (x_1, \ldots, x_n)$, $x_i \in \{0,1\}^t$, to their corresponding receivers using a shared noiseless channel. Receiver $j \in [n] := \{1, 2, \ldots, n\}$ has prior knowledge of a subset $x(A_j) := (x_i \colon i \in A_j)$, $A_j \subseteq [n] \setminus \{j\}$, of the messages and wishes to recover x_j. It is assumed that the sender is aware of $A_1, \ldots, A_n$. The goal is to minimize the amount of information that should be broadcast from the sender to the receivers so that every receiver can recover its desired message.

Originally introduced by Birk and Kol [1], [2] in the context of satellite communication, the index coding problem has received significant attention from various disciplines such as theoretical computer science [3]–[6], information theory [7], [8], network coding [9], [10], and wireless communication [11], [12]. In addition to satellite communication, applications of index coding include peer-to-peer video distribution [5], wireless network interference management [11], [12], and distributed caching [13]. More importantly, index coding is a representative instance of the multiple-unicast network coding problem (see Figure 1), and in fact, every multiple-unicast network coding problem corresponds to an index coding problem [10].

As a shorthand notation, we represent an index coding problem by $(1|A_1), \ldots, (n|A_n)$. It can be also represented by a directed graph $G = (V, E)$, referred to as the side information graph,[1] where $V = [n]$ and $(i, j) \in E$ iff $i \in A_j$; see Figure 1 for an illustration.

[1]Some papers use the opposite convention in which the directions of the edges are reversed.

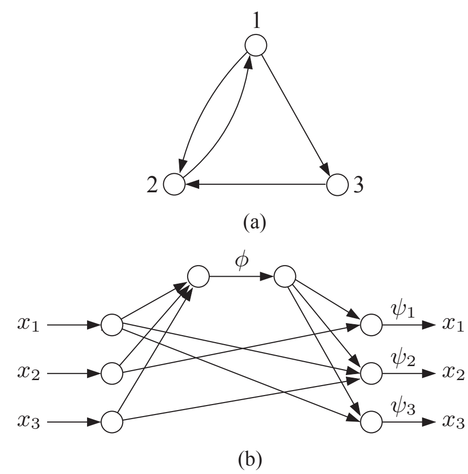

Fig. 1. (a) The graph representation for the index coding problem $(1|2)$, $(2|1,3)$, $(3|1)$. (b) The equivalent network coding problem.

Given an index coding problem, a t-bit *index code* $\mathcal{C} = (\phi, \{\psi_j\})$ is defined by

- an encoder $\phi : \{0,1\}^{nt} \to \{0,1\}^r$ that maps n-tuple of t-bit messages to an r-bit index and
- n decoders, where the jth decoder $\psi_j : \{0,1\}^r \times \{0,1\}^{|A_j|t} \to \{0,1\}^t$ maps the received index $\phi(x^n)$ and the side information $x(A_j)$ to a t-bit message estimate

such that for all $x^n \in \{0,1\}^{nt}$ and all $j \in [n]$,

$$\psi_j(\phi(x^n), x(A_j)) = x_j.$$

The performance of a code $\mathcal{C}$ is measured by its broadcast rate $\beta(\mathcal{C}) = r/t$. The *optimal broadcast rate* of the index coding problem is defined as

$$\beta = \inf_t \inf_{\mathcal{C}} \beta(\mathcal{C}),$$

where the second infimum is over all t-bit index codes. Thus, β characterizes the fundamental limit on the broadcast rate of index codes such that every message can be recovered exactly.

A computable characterization of β is not known in general. Consequently, numerous coding schemes have been proposed in the literature with varying degrees of generality and performance [1]–[8], [14]. Among these, we focus on the coding schemes by Birk and Kol [1], by Blasiak, Kleinberg, and Lubetzky [6] and by Shanmugam, Dimakis, and Langberg [8] that can be viewed

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as time sharing over subproblems of a given index coding problem. As we will review in Section II, these schemes can be implemented rather easily using linear parity-check codes such as maximum distance separable (MDS) codes [15], and the corresponding broadcast rates can be characterized as simple graph-theoretic quantities. In particular, the coding scheme by Shanmugam et al. [8] further reduces the rates needed for time sharing by applying the very idea of index coding to transmission of subproblem indices, which in essence achieves the effect of time sharing "locally" at each receiver over subproblem indices it cannot infer from side information.

Motivated by this local time sharing idea, we explore potential extensions of these coding schemes [1], [6], [8] and study the fundamental limit on such extensions. In Section III, we first combine the key ideas of the existing time-sharing schemes to develop a coding scheme called "fractional local partial clique covering" that is stronger than all of them (Theorem 3). We then develop a coding scheme that recursively decomposes the problem to subproblems and applies local time sharing at every level (Theorem 4). It is shown that this recursive coding scheme strictly improves upon fractional local partial clique covering. Pushing this idea further, we investigate local time sharing over the optimal subproblem broadcast rates and show that this scheme is still suboptimal.

In Section IV, we develop a recursive coding scheme that allows asymmetric message rates for subproblems (Theorem 6) and show that such asymmetric coding strictly improves upon symmetric coding discussed in the previous section. As the culmination of all time-sharing coding schemes previously developed in the literature and newly developed in this paper, we investigate local time sharing over the subproblem capacity regions and the corresponding upper bound on the optimal broadcast rate, which constitutes the fundamental limit of local time sharing for index coding. Rather anticlimactically, this bound is shown to be loose in general. In Section V, we conclude the paper with a discussion on the main deficiency of local time sharing and potential improvements.

Throughout the paper, we use $G|_S = (S, \{(i,j) \in E\colon i,j \in S\})$ to denote the directed subgraph induced by the subset S of the vertices of $G = (V,E)$.

II. Existing Time-Sharing Schemes

The first and simplest approach to index coding is to partition the side information graph G by cliques and transmit the binary sums (parities) of all the messages in each clique. This coding scheme by Birk and Kol [1] achieves a broadcast rate equal to the minimum number of cliques that partition G (or equivalently, the chromatic number of the undirected complement of G) which is the solution to the integer program

$$\begin{aligned} \text{minimize} \quad & \sum_{S \in \mathcal{K}} \rho_S \\ \text{subject to} \quad & \sum_{S \in \mathcal{K}: j \in S} \rho_S \geq 1, \quad j \in [n], \\ & \rho_S \in \{0,1\}, \quad S \in \mathcal{K}, \end{aligned} \tag{1}$$

where $\mathcal{K}$ is the collection of all cliques in G.

This coding scheme, which can be viewed as *time division* over a clique partition (one parity bit per clique), has been extended in several directions. First, Birk and Kol [1] showed that one can use an MDS code over a finite field and perform time division over arbitrary subgraphs (partial cliques) instead of cliques. The number of parity symbols needed for a subgraph H is characterized by the difference $\kappa(H)$ between the number of vertices in H and the minimum indegree within H.

Theorem 1 (Birk and Kol [1])**.** *If $G_1, \ldots, G_m$ partition G, then the optimal broadcast rate is upper bounded by*

$$b_{\mathrm{PC}}(G_1, \ldots, G_m) = \sum_{i=1}^{m} \kappa(G_i) \tag{2}$$

and thus by

$$b_{\mathrm{PC}}(G) = \min_{G_1, \ldots, G_m} b_{\mathrm{PC}}(G_1, \ldots, G_m),$$

where the minimum is over all partitions.

As another extension of (1), Blasiak, Kleinberg, and Lubetzky [6] considered *time sharing* over all cliques (fractional parity bits per clique) so that the combined rate of each message over all parities it participates in is at least one. The resulting rate corresponds to the solution to the linear program obtained by relaxing the integer constraint $\rho_S \in \{0,1\}$ in (1) to $\rho_S \in [0,1]$, which is equivalent to the *fractional chromatic number* of the undirected complement of G.

Shanmugam, Dimakis, and Langberg [8] further extended this scheme by *fractional local clique covering*, whereby an MDS code is applied to parity symbols for cliques. This improves upon the previous time-sharing scheme since each receiver can recover some parity symbols from its side information and thus the total transmission time is now shared only among those parity symbols not available locally at each receiver.

Theorem 2 (Shanmugam, Dimakis, and Langberg [8])**.** *The optimal broadcast rate is upper bounded by the solution $b_{\mathrm{FL}}(G)$ to the linear program*

$$\begin{aligned} \text{minimize} \quad & \max_{j \in [n]} \sum_{S \in \mathcal{K}: S \not\subseteq A_j} \rho_S \\ \text{subject to} \quad & \sum_{S \in \mathcal{K}: j \in S} \rho_S \geq 1, \quad j \in [n], \\ & \rho_S \in [0,1], \quad S \in \mathcal{K}. \end{aligned} \tag{3}$$

The improvement over time sharing is captured by the summation of ρ_S over cliques $S \not\subseteq A_j$ compared to the summation over all cliques S in (1) and can be strict [8]. It can be shown [16] that $b_{\mathrm{FL}}(G)$ in Theorem 2 and $b_{\mathrm{PC}}(G)$ in Theorem 1 are not comparable.

III. Symmetric Coding Schemes

We now develop several extensions of the time-sharing coding schemes in the previous section based on local time sharing. As a first exercise, we combine the ideas of partial clique covering (Theorem 1) and fractional local clique covering (Theorem 2) to establish the following *fractional local partial clique covering* bound.

Theorem 3. *The optimal broadcast rate is upper bounded by the solution $b_{\mathrm{FLP}}(G)$ to the linear program*

$$\begin{aligned} \text{minimize} \quad & \max_{j\in[n]} \sum_{S\subseteq[n]:S\not\subseteq A_j} \rho_S \cdot \kappa(G|_S) \\ \text{subject to} \quad & \sum_{S\subseteq[n]:j\in S} \rho_S \ge 1, \quad j\in[n], \\ & \rho_S \in [0,1], \quad S\subseteq[n]. \end{aligned} \tag{4}$$

Clearly this bound contains partial clique covering and fractional local clique covering bounds as its special cases. The 5-node example in Figure 2 shows that $b_{\mathrm{FLP}}(G)$ is strictly tighter than the minimum of $b_{\mathrm{FL}}(G)$ and $b_{\mathrm{PC}}(G)$.

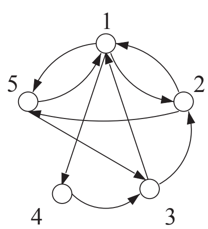

Fig. 2. An index coding problem with $b_{\mathrm{FLP}}(G) = 7/2 < b_{\mathrm{FL}}(G) = b_{\mathrm{PC}}(G) = 4$. Here the rate $b_{\mathrm{PC}}(G) = 4$ is achieved by partitioning the graph into the partial cliques $\{1\}, \{4\}$, and $\{2,3,5\}$. The rate $b_{\mathrm{FL}}(G) = 4$ is achieved by assigning weight 1 to the cliques $\{1,2\}, \{3\}, \{4\}$, and $\{5\}$, and the rate $b_{\mathrm{FLP}}(G) = 7/2$ is achieved by assigning weight $1/2$ to the partial cliques $\{1,2\}, \{1,3,5\}$, and $\{2,3,5\}$, and weight 1 to the partial clique $\{4\}$ (note that we have $\kappa(G|_{\{1,3,5\}}) = \kappa(G|_{\{2,3,5\}}) = 2$). The optimality of each case is verified by solving the respective linear program.

The main idea behind fractional local partial clique covering is to perform local time sharing over all subgraphs. It can be further extended by applying local time sharing recursively for each subgraph.

Theorem 4. *The optimal broadcast rate β of an index coding problem with side information graph G is upper bounded by $b_{\mathrm{R}}(G)$ which is recursively defined as the solution to the linear program*

$$\begin{aligned} \text{minimize} \quad & \max_{j\in[n]} \sum_{S\subsetneq[n]:S\not\subseteq A_j} \rho_S b_{\mathrm{R}}(G|_S) \\ \text{subject to} \quad & \sum_{S\subsetneq[n]:j\in S} \rho_S \ge 1, \quad j\in[n], \\ & \rho_S \in [0,1], \quad S\subsetneq[n], \end{aligned} \tag{5}$$

where $b_{\mathrm{R}}(G|_S)$ is the solution for the subgraph $G|_S$ and $b_{\mathrm{R}}(G|_{\{j\}}) = 1$, $j \in [n]$.

As expected, this bound is tighter than the fractional local partial clique covering bound (Theorem 3).

Theorem 5. $b_{\mathrm{R}}(G) \le b_{\mathrm{FLP}}(G)$.

Proof: First note that for every directed graph G, we have $b_{\mathrm{R}}(G) \le \kappa(G)$. This can be verified by assigning nonzero rates only to subsets of cardinality one. Let $(\rho_S, S\subseteq[n])$ be a feasible solution to (4) such that

$$b_{\mathrm{FLP}}(G) = \max_{j\in[n]} \sum_{S\subseteq[n]:S\not\subseteq A_j} \rho_S \cdot \kappa(G|_S).$$

If $\rho_{[n]} = 0$, $(\rho_S, S \subsetneq [n])$ is feasible to (5) and the proof is complete, since for every subset $S \subsetneq [n]$, $b_{\mathrm{R}}(G|_S) \le \kappa(G|_S)$. If $\rho_{[n]} > 0$, define

$$\rho'_S = \begin{cases} 0, & \text{if } S = [n], \\ \min\{\rho_S + \rho_{[n]}, 1\}, & \text{if } |S| = 1, \\ \rho_S, & \text{otherwise.} \end{cases}$$

Then it can be checked that $(\rho'_S, S\subsetneq[n])$ is a feasible solution to (5). In addition, for all $j\in[n]$,

$$\begin{aligned} & \sum_{S\subsetneq[n]:S\not\subseteq A_j} \rho'_S b_{\mathrm{R}}(G|_S) \\ &= \sum_{S\subsetneq[n]:S\not\subseteq A_j} \rho_S b_{\mathrm{R}}(G|_S) + \sum_{k\in[n]:k\notin A_j} \rho_{[n]} b_{\mathrm{R}}(G|_{\{k\}}) \\ &\le \sum_{S\subsetneq[n]:S\not\subseteq A_j} \rho_S \cdot \kappa(G|_S) + \rho_{[n]} \cdot \kappa(G) \\ &= \sum_{S\subseteq[n]:S\not\subseteq A_j} \rho_S \cdot \kappa(G|_S). \end{aligned}$$

Therefore,

$$\begin{aligned} b_{\mathrm{R}}(G) &\le \max_{j\in[n]} \sum_{S\subsetneq[n]:S\not\subseteq A_j} \rho'_S b_{\mathrm{R}}(G|_S) \\ &\le \max_{j\in[n]} \sum_{S\subseteq[n]:S\not\subseteq A_j} \rho_S \cdot \kappa(G|_S) = b_{\mathrm{FLP}}(G). \quad \blacksquare \end{aligned}$$

The inequality in Theorem 5 is sometimes strict, as demonstrated by the 5-node problem in Figure 3.

Remark 1. Both $b_{\mathrm{R}}(G)$ and $b_{\mathrm{FLP}}(G)$ can be shown to be tight for all index coding problems with up to 4 messages.

The upper bound $b_{\mathrm{R}}(G)$ is, however, not tight in general. In fact, we can use the optimal index code for every subproblem $S \subsetneq [n]$, or equivalently, we can replace $b_{\mathrm{R}}(G|_S)$ with $\beta(G|_S)$ in (5). This establishes an even

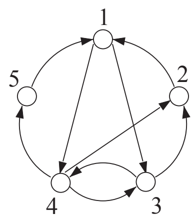

Fig. 3. An index coding problem with $b_{\mathrm{R}}(G) = 3 < b_{\mathrm{FLP}}(G) = 7/2$. The rate $b_{\mathrm{R}}(G) = 3$ is achieved by assigning weight 1 to subsets $\{1, 2, 3, 4\}$ and $\{5\}$ (note that $\beta(G|_{\{1,2,3,4\}}) = 2$, achieved by composite coding [14]). The rate $b_{\mathrm{FLP}}(G) = 7/2$ is achieved by assigning weight $1/2$ to the partial cliques $\{1, 2, 3\}, \{1, 2, 4\}$, and $\{3, 4\}$ and weight 1 to the partial clique yields $b_{\mathrm{FLP}}(G) = 7/2$ (note that $\kappa(G|_{\{1,2,3\}}) = \kappa(G|_{\{1,2,4\}}) = 2$). The optimality of each case is verified by solving the respective linear program.

tighter bound $b_{\mathrm{LTS}}(G)$ achieved by local time sharing between the optimal broadcast rates of the subgraphs. A natural question arises: whether $b_{\mathrm{LTS}}(G)$ is tight. The following example shows that the answer to this question is negative.

Example 1. Consider the 7-node index coding problem

$$(1|2,3,4,6,7),\ (2|1,3,6,7),\ (3|1,4,5,7),\ (4|1,2,5,6),$$
$$(5|3,4,6,7),\ (6|2,4,5,7),\ (7|2,3,5,6).$$

For this example, we have $\beta = 5/2 < b_{\mathrm{LTS}}(G) = 8/3$, with β achieved by subspace interference alignment [11].

One reason behind the suboptimality of local time sharing over optimal broadcast rates could be that the fractional message rates for each subproblem are unnecessarily restricted to be identical. Therefore, one way to improve $b_{\mathrm{LTS}}(G)$ could be to consider the entire tradeoff between the fractional message rates in the subproblems, which we investigate in the next section.

IV. Asymmetric Coding Schemes

We first extend our definition of the index coding problem in Section I to the case of asymmetric message rates [6], in which the sender wishes to communicate n messages $x_j \in \{0, 1\}^{tR_j}$, $j \in [n]$, by broadcasting t bits. The goal is to characterize the optimal tradeoff between the individual rates $R_j, j \in [n]$, that are the reciprocals of the individual broadcast rates β_j.

Given an index coding problem, a $(2^{tR_1}, \ldots, 2^{tR_n}, t)$ *index code* $\mathcal{C} = (\phi, \{\psi_j\})$ is defined by

- an encoder $\phi : \{0, 1\}^{\sum_{j=1}^{n} tR_j} \to \{0, 1\}^t$ that maps n-tuple of messages to a t-bit index, and
- n decoders, where the jth decoder $\psi_j : \{0, 1\}^t \times \{0, 1\}^{\sum_{k\in A_j} tR_k} \to \{0, 1\}^{tR_j}$ maps the received string $y^t = \phi(x_1, \ldots, x_n)$ and the side information $x(A_j)$ to a tR_j-bit message estimate,

such that for all $x^n \in \{0, 1\}^{\sum_{j=1}^{n} tR_j}$, we have $\psi_j(\phi(x^n), x(A_j)) = x_j$ for all $j \in [n]$. A rate tuple $(R_1, \ldots, R_n)$ is said to be achievable if there exists a $(2^{tR_1}, \ldots, 2^{tR_n}, t)$ index code $\mathcal{C}$. The capacity region $\mathscr{C}$ of the index coding problem is the closure of the set of achievable rate tuples $(R_1, \ldots, R_n)$. The following relation holds between the broadcast rate characterization of the problem and the capacity region characterization:

$$\beta = \beta(\mathscr{C}) := \min\{(1/R) \colon (R, \ldots, R) \in \mathscr{C}\}.$$

Continuing our program of local time sharing in the previous section, we establish the following recursive inner bound on the capacity region.

Theorem 6. *The capacity region $\mathscr{C}$ of the index coding problem with side information graph G contains the rate region $\mathscr{R}_{\mathrm{R}}(G)$ that is recursively defined as the set of rate tuples $(R_1, \ldots, R_n)$ such that*

$$R_j = \sum_{S \subsetneq [n]} T_{j,S}, \quad j \in [n], \tag{6}$$

for some $(T_{j,S} \colon j \in S)$ and γ_S, $S \subsetneq [n]$, satisfying

$$\begin{aligned} \sum_{S \subsetneq [n] : S \not\subseteq A_j} \gamma_S &\le 1, \quad j \in [n], \\ (T_{j,S} \colon j \in S) &\in \gamma_S \cdot \mathscr{R}_{\mathrm{R}}(G|_S), \quad S \subsetneq [n], \qquad (7) \\ \gamma_S &\ge 0, \quad S \subsetneq [n], \\ T_{j,S} &\ge 0, \quad S \subsetneq [n], j \in S, \end{aligned}$$

where $\mathscr{R}_{\mathrm{R}}(G|_S)$ is the rate region for the subgraph $G|_S$ and $\mathscr{R}_{\mathrm{R}}(G|_{\{j\}}) = [0, 1]$. Here, $a \cdot \mathscr{R} := \{aR \colon R \in \mathscr{R}\}$.

Let $b(\mathscr{R}_{\mathrm{R}}(G)) = \min\{(1/R) \colon (R, \ldots, R) \in \mathscr{R}_{\mathrm{R}}(G)\}$ be the minimum broadcast rate associated with $\mathscr{R}_{\mathrm{R}}(G)$. Without symmetric rate constraints on subproblems, recursion over rate regions (Theorem 6) is richer than recursion over broadcast rates (Theorem 4). The following proposition confirms this intuition formally.

Proposition 1. *For the index coding problem with side information graph G, we have*

$$b(\mathscr{R}_{\mathrm{R}}(G)) \le b_{\mathrm{R}}(G). \tag{8}$$

Proof: We use induction on the number n of messages. The induction base is trivially true. Assume that (8) holds for all index coding problems with $n - 1$ or less messages. Let $(\rho_S, S \subsetneq [n])$ be a feasible solution to (5) such that

$$b_{\mathrm{R}}(G) = \max_{j\in[n]} \sum_{S \subsetneq [n] : S \not\subseteq A_j} \rho_S b_{\mathrm{R}}(G|_S).$$

For all $S \subsetneq [n]$, define

$$\gamma_S = \frac{\rho_S b_{\mathrm{R}}(G|_S)}{b_{\mathrm{R}}(G)} \quad \text{and} \quad T_{j,S} = \begin{cases} \frac{\rho_S}{b_{\mathrm{R}}(G)} & \text{if } j \in S, \\ 0 & \text{otherwise.} \end{cases}$$

Then, for all $j \in [n]$ we have

$$\sum_{S \subsetneq [n] : S \not\subseteq A_j} \gamma_S = \sum_{S \subsetneq [n] : S \not\subseteq A_j} \frac{\rho_S b_{\mathrm{R}}(G|_S)}{b_{\mathrm{R}}(G)} \le 1, \tag{9}$$

$$R_j = \sum_{S \subsetneq [n]} T_{j,S} = \sum_{S \subsetneq [n]: j \in S} \frac{\rho_S}{b_{\mathrm{R}}(G)} \geq \frac{1}{b_{\mathrm{R}}(G)}. \quad (10)$$

In addition, since $(1/b_{\mathrm{R}}(G|_S), \ldots, 1/b_{\mathrm{R}}(G|_S)) \in \mathscr{R}_{\mathrm{R}}(G|_S)$ and $\gamma_S / b_{\mathrm{R}}(G|_S) = \rho_S / b_{\mathrm{R}}(G)$, by the induction hypothesis, we have $(T_{j,S} : j \in S) \in \gamma_S \cdot \mathscr{R}_{\mathrm{R}}(G|_S)$, which completes the proof. ■

However, The inner bound $\mathscr{R}_{\mathrm{R}}(G)$ and the associated broadcast rate $b(\mathscr{R}_{\mathrm{R}}(G))$ are not tight in general and can be improved by using the capacity region $\mathscr{C}(G|_S)$ of the subgraph $G|_S$ instead of $\mathscr{R}_{\mathrm{R}}(G|_S)$ in (6). This establishes a tighter inner bound $\mathscr{R}_{\mathrm{LTS}}(G)$ and the associated broadcast rate $b(\mathscr{R}_{\mathrm{LTS}}(G))$, the latter of which is the tightest upper bound on the optimal broadcast rate achieved by local time sharing.

Following similar steps to the proof of Proposition 1, we can establish:

Proposition 2. $b(\mathscr{R}_{\mathrm{LTS}}(G)) \leq b_{\mathrm{LTS}}(G)$.

This inequality can be strict. For the 7-node index coding problem in Example 1, solving the corresponding linear programs yields $\beta = b(\mathscr{R}_{\mathrm{LTS}}(G)) = 5/2 < b_{\mathrm{LTS}}(G) = 8/3$. This shows that asymmetric coding of subproblems is necessary even when the problem is on the optimal symmetric rate.

Now the natural question becomes whether $\mathscr{R}_{\mathrm{LTS}}(G)$ and $b(\mathscr{R}_{\mathrm{LTS}}(G))$ are tight in general. The 5-node example in Figure 4 shows that local time sharing over the capacity regions of subproblems yields neither the capacity region nor the optimal broadcast rate.

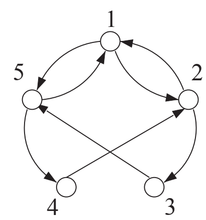

Fig. 4. An index coding problem with $\beta = 3 < b(\mathscr{R}_{\mathrm{LTS}}(G)) = b_{\mathrm{LTS}}(G) = 7/2$. Here β is achieved by composite coding [14].

V. Discussion

In this paper, we considered the class of index coding schemes based on time sharing and presented a series of extensions of the existing schemes in this class. It is shown that even the best scheme in this class, namely, local time sharing between capacity regions of subproblems, is not optimal, which demonstrates a fundamental limitation of the concept of local time sharing.

The main performance bottleneck (and the main culprit behind the suboptimality) is that in local time sharing, each receiver is required to recover all subproblem indices, even those that are not needed to recover its own message.[2] Hence, relaxing this unnatural requirement would be crucial to improve upon local time sharing, but any immediate solution seems to be tantamount to solving an index coding problem with a larger number of messages.

Another weakness of local time sharing could be that, through explicit rate splitting, the problem is divided into a set of subproblems that have no interaction among themselves. In comparison, composite coding [14] utilizes all relevant composite indices simultaneously to encode and decode. (Note that neither composite coding nor local time sharing over capacity regions of subproblems outperform the other scheme.) Vector linear coding (or more specifically interference alignment [7]) takes a more holistic approach, that leads to a globally better linear solution (which is in general very difficult to find).

[2]Note that a similar performance bottleneck also arises in the composite coding scheme for recovering composite indices [14].

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