Capacity Scaling for Cloud Radio Access Networks with Limited Orthogonal Fronthaul

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***Abstract*—Uplink and downlink cloud radio access networks are modeled as two-hop K-user L-relay networks, whereby small base-stations act as relays and are connected to a central processor via orthogonal fronthaul links of finite capacities. Based on noisy network coding and distributed decode–forward inner bounds on the capacity regions for uplink and downlink, respectively, the total fronthaul link capacity required to approach the centralized MIMO sum-rate is characterized. The capacity scaling law when the network size increases is examined under certain uplink and downlink network models, both theoretically and via simulations.**

I. Introduction

With ever-increasing demands for higher data rates, better coverage, and reliability of communication for a large number of devices, novel network protocols and architectures are expected to play an important role in future communication systems. The cloud radio access network (C-RAN) architecture [1], [2] is one of the promising candidates, in which communication over a group of cells is coordinated by a cloud-based central processor.

Base stations in a C-RAN, unlike in conventional cellular networks, do not perform all network functionalities locally, but instead delegate most of them to a central processor by communicating with it over wired or wireless fronthaul links. If these links have unbounded capacities, the C-RAN can be viewed as a "distributed" multiple input multiple output (MIMO) system, where base stations act as spatially distributed antennas that use beamforming to coordinate transmission and mitigate interference among multiple cells. For the more realistic situation of limited fronthaul link capacities, the optimal beamforming solution is typically computed without the capacity constraints. The corresponding baseband signals are then digitized individually for each base station and transmitted to (or from) the central processor through the fronthaul links, which often leads to high fronthaul capacity requirements.

As an alternative to this greedy beamforming–digitization approach, this paper investigates near-optimal coding schemes for the C-RAN architecture and their achievable throughput tradeoffs by modeling the entire system as a two-hop relay network. In this model, which was studied, for example, in [1], [3], [4], [5], the base stations act as relays that summarize the received signals from user devices to the central processor (uplink) and transmit the prescribed signals from the central processor to user devices (downlink). Communication-theoretic results on this model were presented in a recent volume edited by Quek, Peng, Simeone, and Yu [1].

In this paper, we further study the inner bounds for uplink and downlink C-RAN in the literature (see, for example, [3], [4], [5], [6], [7], [8], [9], [10], [11], and references therein) and investigate the question of how much fronthaul link capacities are needed for the C-RAN to emulate the centralized MIMO system. Studying the sum-capacity scaling for uplink and downlink C-RAN in the large network size limit under various network models, we demonstrate that we can achieve similar scaling as centralized MIMO uplink, provided we have a slightly larger total fronthaul capacity at our disposal.

The rest of the paper is organized as follows. Section II studies the uplink C-RAN. Section II-A briefly describes the uplink network model and introduces an inner bound which approaches the centralized MIMO capacity region as the fronthaul capacity becomes infinite; Section II-B establishes the fronthaul capacity required to approximate the centralized MIMO uplink sum-capacity; and Section II-C uses the results of Section II-B to examine the sum-capacity scaling for uplink C-RAN under various network models. Section III studies the downlink C-RAN. Section III-A describes the downlink network model, presents an inner bound, and quantifies the fronthaul capacity required to approximate the centralized MIMO downlink sum-capacity. Section III-B examines the capacity scaling for downlink C-RAN. Throughout the paper, we follow the notation in [12]. In addition, we use the following. For a natural number n, we denote by $[n]$ the set $\{1, \dots, n\}$. For functions f and g from $\mathbb{R}$ to $\mathbb{R}$, we say $f \sim g$ if $f(x)/g(x) \to 1$ as $x \to \infty$. All logarithms are to base 2 and all information measures are in bits.

II. Uplink Communication

A. Network Model and the Capacity Inner Bound

We model the uplink C-RAN as a two-hop relay network in Fig. 1. The first hop, namely, the (wireless) channel from the user devices to the radio heads, is modeled as a memoryless Gaussian network $Y^L = GX^K + Z^L$, where X^K is the channel input vector from K users, each with average power constraint P, $G \in \mathbb{R}^{L\times K}$ is a (deterministic) channel gain matrix, and Z^L is a vector of independent $\mathrm{N}(0,1)$ noise components. The second hop, namely, the channel from the radio heads to the central processor, consists of orthogonal

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links of capacities $C_1, \dots, C_L$ bits per transmission, decoupled from the first hop. To be more precise, the channel output at the central processor (receiver) is $(W_1, \dots, W_L)$, where $W_l \in [2^{nC_l}]$ denotes the message relay l communicates to the receiver. A $(2^{nR_1}, \dots, 2^{nR_K}, n)$ code for this network, the average probability of error for a code, the achievability of a rate tuple $(R_1, \dots, R_K)$, and the capacity region are defined in the standard fashion; see, for example, [5], [12].

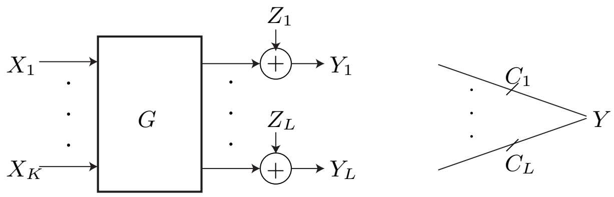


Fig. 1: Uplink network model.

Sanderovich et al. [6] specialized the network compress–forward scheme [7] to this network. Roughly speaking, each relay compresses its received sequence and sends the bin index of the compression index. This coding scheme yields the following inner bound on the capacity region of the C-RAN uplink.

Proposition 1. *A rate tuple $(R_1, \dots, R_K)$ is achievable if*

$$\sum_{k \in S_1} R_k < \frac{1}{2} \log \left| \frac{P}{\sigma^2 + 1} G_{S_2^c, S_1} G_{S_2^c, S_1}^T + I \right| + \sum_{l \in S_2} C_l - \frac{|S_2|}{2} \log \left(1 + \frac{1}{\sigma^2} \right), \tag{1}$$

for all $S_1 \subseteq [K]$ and $S_2 \subseteq [L]$ for some $\sigma^2 > 0$, where $G_{S_2^c, S_1}$ is the submatrix of G formed by the rows with indices in S_2^c and the columns with indices in S_1.

The same inner bound can also be achieved by specializing the noisy network coding scheme [13], as demonstrated in [5]. For a fixed $\sigma^2 > 0$, we denote the region defined by (1) as $\mathscr{R}_{\text{up}}(\sigma^2)$.

Remark 1. As the fronthaul capacities $C_1, \dots, C_L \to \infty$, the inner bound (1) converges to the capacity region of the centralized MIMO uplink with K senders and a receiver with L receive antennas. Intuitively, having infinite fronthaul is equivalent to the relays being co-located with the receiver.

B. Comparison with Centralized MIMO Uplink

Unfortunately, for *finite* fronthaul link capacities $C_1, \dots, C_L$, no matter how large they are, we can always find networks for which the capacity region of the uplink C-RAN is strictly less than the centralized MIMO MAC capacity region, as demonstrated by the following example.

Example 1. Consider the one-sender, two-relay Gaussian uplink C-RAN with first hop

$$p_{Y_1, Y_2|X}(y_1, y_2|x) \equiv p_{Y|X}(y_1|x) p_{Y|X}(y_2|x)$$

and fronthaul capacities $C_1 = C$ and $C_2 = \infty$. In other words, the MAC in the first hop has conditionally i.i.d. outputs Y_1, Y_2 given X. Then, this network corresponds to the Gaussian version of Cover's problem [14] and by the results of [15], the capacity of this network for any finite C is strictly less than the capacity for $C = \infty$.

On the positive side, it is possible to approximately achieve the centralized MIMO uplink sum-rate for finite fronthaul capacities, provided we spend a certain amount of extra capacity on the fronthaul. To show this, we need the following.

Definition 1. An N-dimensional *polymatroid* is a subset of $[0, \infty)^N$ that can be expressed in the form

$$\mathscr{P}(\phi) = \Big\{ (x_1, \dots, x_N) \subseteq \mathbb{R}_+^N : \sum_{l \in S} x_l \le \phi(S), S \subseteq [N] \Big\},$$

where $\phi : 2^{[N]} \to \mathbb{R}_+ := [0, \infty)$ is a set function satisfying $\phi(\emptyset) = 0$, $\phi(S) \le \phi(T)$ if $S \subseteq T$, and $\phi(S \cup T) + \phi(S \cap T) \le \phi(S) + \phi(T)$.

The following result [16] will be useful in rewriting the achievable sum-rate for the Gaussian uplink C-RAN in an alternative form.

Edmonds's Polymatroid Intersection Theorem. *If $\mathscr{P}(\phi)$ and $\mathscr{P}(\psi)$ are two polymatroids, then*

$$\max \left\{ \sum_{l \in [N]} x_l : (x_1, \dots, x_N) \in \mathscr{P}(\phi) \cap \mathscr{P}(\psi) \right\} = \min_{S \subseteq [N]} \left(\phi(S) + \psi(S^c) \right).$$

Using this, we can quantify the fronthaul requirement for noisy network coding to approximate the centralized MIMO MAC capacity as follows.

Theorem 1. *If*

$$C^* \ge \frac{1}{2} \log \left| \frac{P}{\sigma^2 + 1} G G^T + I \right| + \frac{L}{2} \log \left(1 + \frac{1}{\sigma^2} \right)$$

for some $\sigma^2 > 0$, then there exist fronthaul link capacities $C_1, C_2, \dots, C_L \ge 0$ with $\sum_{l \in [L]} C_l = C^*$ at which noisy network coding can achieve a sum-rate

$$R_{\text{sum}}(\text{NNC}) = \frac{1}{2} \log \left| \frac{P}{\sigma^2 + 1} G G^T + I \right|.$$

Conversely, to achieve the centralized MIMO sum-rate $R_{\text{sum}}(\text{MIMO}) = (1/2) \log |I + PGG^T|$, we must have a total fronthaul capacity

$$C^* \ge \frac{1}{2} \log |I + PGG^T|.$$

Proof: The maximum sum-rate corresponding to $\mathscr{R}_{\text{up}}(\sigma^2)$ can be written as

$$\begin{aligned} R_{\text{sum}}(\text{NNC}) &= \min_{S_2 \subseteq [L]} \Big(\frac{1}{2} \log \Big| \frac{P}{\sigma^2 + 1} G_{S_2^c, [K]} G_{S_2^c, [K]}^T + I \Big| \\ &\qquad + \sum_{l \in S_2} C_l - \frac{|S_2|}{2} \log \big(1 + \frac{1}{\sigma^2} \big) \Big) \\ &= \min_{S_2 \subseteq [L]} \Big(\phi(S_2^c) + \psi(S_2) \Big), \end{aligned} \tag{2}$$

where

$$\phi(S_2^c) := \frac{1}{2}\log\left|\frac{P}{\sigma^2+1}G_{S_2^c,[K]}G_{S_2^c,[K]}^T + I\right|,$$

$$\psi(S_2) := \sum_{l\in S_2}\left(C_l - \frac{1}{2}\log\left(1+\frac{1}{\sigma^2}\right)\right)$$

can be easily verified to be such that $\mathscr{P}(\phi)$ and $\mathscr{P}(\psi)$ are both polymatroids whenever $C_l \geq (1/2)\log(1+1/\sigma^2)$ for all $l\in[L]$. Therefore, by Edmonds's polymatroid intersection theorem, $R_{\text{sum}}(\text{NNC}) = \max\{\sum_{l\in[L]} y_l\}$, where the maximum is over all tuples $(y_1,\ldots,y_l)$ satisfying

$$y_l \leq C_l - \frac{1}{2}\log\left(1+\frac{1}{\sigma^2}\right)$$

for all $l\in[L]$ and

$$\sum_{l\in S_2} y_l \leq \phi(S_2)$$

for all $S_2 \subseteq [L]$. Using this alternative representation of $R_{\text{sum}}(\text{NNC})$, it can be shown that a choice of $(C_1,\ldots,C_L)$ satisfying $C_1+\cdots+C_L = C^*$ and guaranteeing $R_{\text{sum}}(\text{NNC}) = \phi([L])$ exists whenever $\psi([L]) \geq \phi([L])$. Thus, the sufficient condition boils down to

$$\begin{aligned}
C^* &= \sum_{l\in[L]}\left(C_l - \frac{1}{2}\log\left(1+\frac{1}{\sigma^2}\right)\right) + \frac{L}{2}\log\left(1+\frac{1}{\sigma^2}\right)\\
&= \psi([L]) + \frac{L}{2}\log\left(1+\frac{1}{\sigma^2}\right)\\
&\geq \phi([L]) + \frac{L}{2}\log\left(1+\frac{1}{\sigma^2}\right)\\
&= \frac{1}{2}\log\left|\frac{P}{\sigma^2+1}GG^T + I\right| + \frac{L}{2}\log\left(1+\frac{1}{\sigma^2}\right),
\end{aligned}$$

establishing the result. The converse follows immediately from the cutset bound. ■

Given C^*, one can come up with an allocation $(C_1,\ldots,C_L)$ which achieves the sum-rate predicted by Theorem 1 by solving a linear feasibility problem involving 2^L-2 inequalities and one equality. One can also modify this feasibility problem to include other constraints such as requiring all but one of the link capacities to be zero. Theorem 1 demonstrates that we can approximate the centralized MIMO uplink sum-capacity, provided we have at our disposal a total fronthaul capacity "slightly" larger than this amount. In the next section, we examine how these sum-capacities scale with network size for specific network models, in order to further examine how closely the achievable sum-rates for the uplink C-RAN approximate the centralized MIMO uplink sum-capacity, and what cost (in terms of fronthaul requirement).

C. Capacity Scaling

1) Rich scattering: We first consider a *rich scattering* network model with slow fading, where the entries of the channel matrix G are i.i.d. $\mathrm{N}(0,1)$ random variables and are assumed fixed for the duration of transmission. Moreover, the average power constraint P is kept fixed. We recall the following.

Lemma 1 (Telatar [17], Silverstein [18])**.** *Let H be an $m\times n$ random matrix with i.i.d. entries H_{ij}, each of which has unit variance. Then, as $n\to\infty$ such that $n/m \to \rho \geq 1$, the limiting density of the empirical distribution of the eigenvalues of HH^T/m is given, almost surely, by*

$$f_\Lambda(\lambda) = \frac{\sqrt{(\lambda-\alpha(\rho))(\beta(\rho)-\lambda)}}{2\pi\lambda}\mathbf{1}_{[\alpha(\rho),\beta(\rho)]},$$

where $\alpha(\rho) := (\sqrt{\rho}-1)^2$ and $\beta(\rho) := (\sqrt{\rho}+1)^2$.

Now, let $L\to\infty$ such that $L/K \to \rho \in (1,\infty]$. By Lemma 1, for every $\epsilon > 0$,

$$\begin{aligned}
&\frac{1}{2}\log|I+PGG^T|\\
&= \frac{1}{2}\log|I+PG^TG|\\
&\in \Big[\frac{K}{2}\log\left(1+PK(\sqrt{\rho}-1)^2\right)-\epsilon,\\
&\qquad \frac{K}{2}\log\left(1+PK(\sqrt{\rho}+1)^2\right)+\epsilon\Big]
\end{aligned}$$

for L sufficiently large, w.p. 1. Therefore,

$$\frac{1}{2}\log|I+PGG^T| \sim \frac{K}{2}\log L \quad \text{w.p. 1.} \tag{3}$$

Similarly, for a fixed $\sigma^2 > 0$, we have

$$\frac{1}{2}\log\left|I+\frac{P}{\sigma^2+1}GG^T\right| \sim \frac{K}{2}\log L \quad \text{w.p. 1.} \tag{4}$$

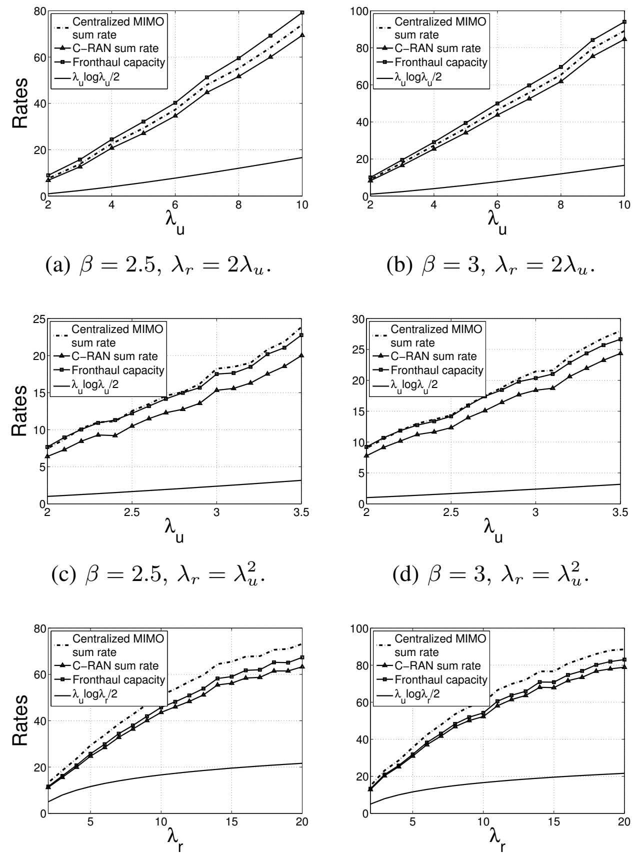
We now consider various scaling regimes of L and K, namely, K fixed and L growing, $L=\gamma K$ with $\gamma>1$, and $L=K^\gamma$ with $\gamma>1$. For each case, we can choose σ^2 appropriately, and use Theorem 1 and Lemma 1 to examine the scaling of the achievable sum-rate and the corresponding C^* required. The results are summarized in Table I. These are limiting results for large network size that hold w.p. 1 under the rich scattering network model. As an example of how to interpret the table, let $L=\gamma K$ with a fixed $\gamma>1$. Then, as $K\to\infty$ with $C^* \sim K\log K/2$, noisy network coding can achieve $R_{\text{sum}}(\text{NNC}) \sim K\log K/2$ w.p. 1 if we choose $\sigma^2 = 1$. We also note that the centralized MIMO uplink capacity scales as $K\log L/2$ in all these cases considered. If K grows faster than L, one can exchange the roles of L and K in Lemma 1 and choose $\sigma^2=1$ to demonstrate that for $L=\gamma K$ with $\gamma<1$, $L=K^\gamma$ with $\gamma<1$, as well as for L fixed and K growing, $R_{\text{sum}}(\text{NNC})$, C^*, and the centralized MIMO sum-rate all grow as $L\log K/2$. For $L=K\to\infty$, Lemma 1 unfortunately does not allow us to make an a.s. statement directly. However, one can choose to only serve $(1-\epsilon)K$ of the users, thereby leading to a sum-rate scaling of $(1-\epsilon)K\log K/2$ in accordance with table I.

2) Stochastic geometry: We now consider a more realistic network model based on stochastic geometry [19]. In this model, users and relays are distributed over a square with unit area according to independent Poisson point processes

L vs. K	σ^2	C^*	$R_{\text{sum}}(\text{NNC})$	$R_{\text{sum}}(\text{MIMO})$
$L = \gamma K$	1	$K \log K/2$	$K \log K/2$	$K \log K/2$
$L = K^\gamma$	$K^{\gamma-1}$ $K^{\gamma-1-\delta}$	$K \log K/2$ $(\log e/2)K^{1+\delta}$	$K \log K/2$ $(1+\delta)K \log K/2$	$\gamma K \log K/2$
K fixed	L^ϵ	$(\log e/2)L^{1-\epsilon}$	$(1-\epsilon)K \log L/2$	$K \log L/2$

TABLE I: Sum rate scaling for uplink; $\gamma > 1$, $0 < \delta < \gamma - 1$, $0 < \epsilon < 1$.

with intensities λ_u and λ_r, respectively. All channel gains are assumed to be real and unchanged for the duration of transmission, and the gain g_{lk} from sender k to relay l, separated by Euclidean distance r, is given by $r^{-\beta}$, where β is the *path loss exponent*. Since a unit area is considered, λ_u and λ_r act as stand-ins for K and L, respectively. We therefore scale λ_u and λ_r in a similar fashion as Section II-C1. In particular, we examine the cases $\lambda_r = 2\lambda_u$, $\lambda_r = \lambda_u^2$, and λ_u fixed. The corresponding median sum-rates for centralized MIMO and C-RAN uplink (with appropriate choices of σ^2 according to Table I), as well as the corresponding C^* required, are plotted as functions of λ_u in Fig. 2 for different values of β. The median values are taken over 1000 runs of the simulations. From the plots, we observe that the uplink C-RAN sum-capacity scales in a similar fashion as the centralized MIMO MAC sum-capacity, provided we have a similar amount to spend on the fronthaul. Under this network model, the C-RAN capacity seems to show a faster scaling than the theoretical one obtained for the rich scattering situation in Section II-C1, the latter having been plotted in all the figures with a solid line. A plausible reason for this is that in the path loss regime $\beta > 2$, the locality of propagation (fast decay of channel gains with distance) helps the capacity by reducing inter-user interference.



(a) $\beta = 2.5$, $\lambda_r = 2\lambda_u$.
(b) $\beta = 3$, $\lambda_r = 2\lambda_u$.
(c) $\beta = 2.5$, $\lambda_r = \lambda_u^2$.
(d) $\beta = 3$, $\lambda_r = \lambda_u^2$.
(e) $\beta = 2.5$, λ_u fixed.
(f) $\beta = 3$, λ_u fixed.

Fig. 2: Uplink capacity scaling under stochastic geometry.

III. Downlink Communication

A. Network Model and the Capacity Inner Bound

Similar to the uplink, we model the downlink C-RAN as a two-hop relay network in Fig. 3, where the first hop (central processor to radio heads) consists of orthogonal noiseless links of capacities $C_1, \ldots, C_L$ bits per transmission and the second hop (radio heads to user devices) is modeled as a memoryless Gaussian network $Y^K = HX^L + Z^K$. Here, $H \in \mathbb{R}^{K \times L}$ is a channel gain matrix and Z^K consists of i.i.d. $\mathrm{N}(0, 1)$ noise components; we assume the average power constraint P at each relay.

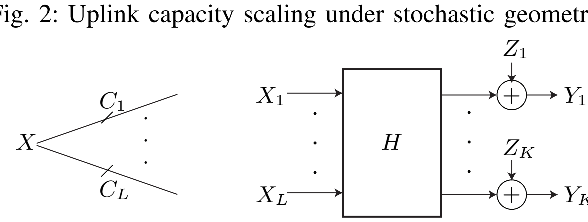


Fig. 3: Downlink network model.

We have the following inner bound on the capacity region of this network, which was obtained in [5] by specializing and simplifying the distributed decode–forward scheme [20].

Proposition 2. *A rate tuple* $(R_1, \ldots, R_K)$ *is achievable if*

$$\sum_{k \in S_2^c} R_k < \frac{1}{2} \log \left| \frac{P}{\sigma^2} H_{S_2^c, S_1} H_{S_2^c, S_1}^T + I \right| + \sum_{l \in S_1^c} C_l - \frac{|S_2^c|}{2} \log \left(1 + \frac{1}{\sigma^2} \right) \qquad (5)$$

for all $S_1 \subseteq [L]$ *and* $S_2 \subseteq [K]$ *for some* $\sigma^2 > 0$.

For a fixed $\sigma^2 > 0$, we denote the region defined by (5) as $\mathscr{R}_{\text{down}}(\sigma^2)$. The maximum sum-rate corresponding to $\mathscr{R}_{\text{down}}(\sigma^2)$ is given by

$$R_{\text{sum}}(\text{DDF}) = \min_{S_1 \subseteq [L]} \Big(\frac{1}{2} \log \left| \frac{P}{\sigma^2} H_{[K], S_1} H_{[K], S_1}^T + I \right| + \sum_{l \in S_1^c} C_l - \frac{K}{2} \log\big(1 + \frac{1}{\sigma^2}\big) \Big).$$

The centralized MIMO downlink sum-capacity is, on the other hand, *upper-bounded* by $(1/2)\log|I + PHH^T|$, corresponding to full cooperation among receivers. Similar to Section II-B, we can now use Edmonds's polymatroid intersection theorem to quantify the total fronthaul required to approximate the centralized MIMO downlink sum-capacity.

Theorem 2. *If*

$$C^* \geq \frac{1}{2}\log\left|\frac{P}{\sigma^2}HH^T + I\right|$$

for some $\sigma^2 > 0$, *then there exist* $C_1, C_2, \ldots, C_L \geq 0$ *with* $\sum_{l\in[L]} C_l = C^*$, *at which distributed decode–forward can achieve a sum-rate*

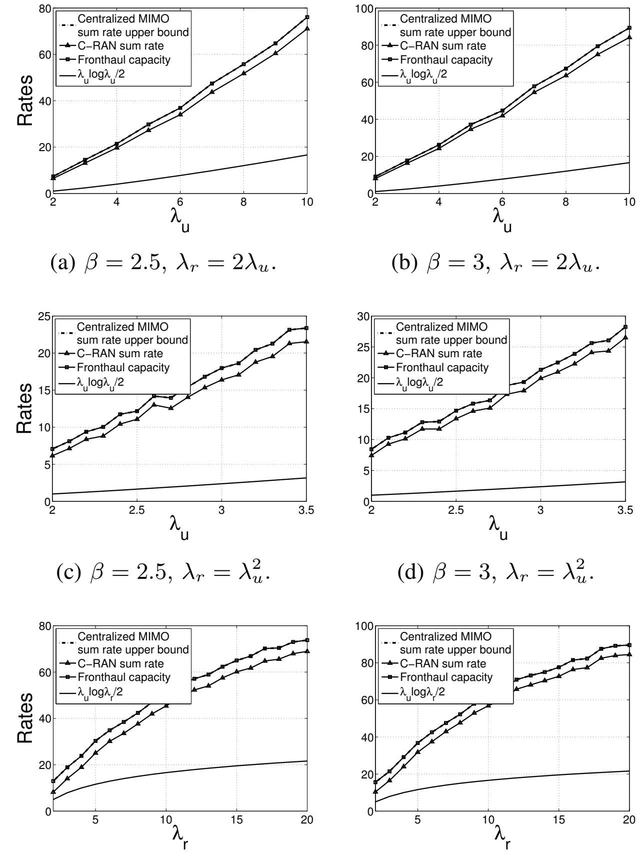
$$R_{\mathrm{sum}}(\mathrm{DDF}) = \frac{1}{2}\log\left|\frac{P}{\sigma^2}HH^T + I\right| - \frac{K}{2}\log\left(1 + \frac{1}{\sigma^2}\right).$$

Conversely, to achieve a sum-rate of $(1/2)\log|I + PHH^T|$, *we must have*

$$C^* \geq \frac{1}{2}\log|I + PGG^T|.$$

B. Capacity Scaling

Similar to Section II-C1, we first consider a rich scattering model and use Lemma 1 and Theorem 2 with $\sigma^2 = 1$. In this case, for all the scaling regimes considered, namely, fixed K, $L = \gamma K$, and $L = K^\gamma$ (the latter two for $\gamma > 1$), the centralized MIMO downlink sum-rate and the downlink C-RAN sum-rate, as well as the fronthaul requirement C^*, show the same scaling $K \log L/2$. For a stochastic geometry model similar to that in Section II-C2, Fig. 4 plots the median sum-rates obtained experimentally over 1000 simulation runs each, for different scaling regimes and different path loss exponents. The power constraint P at each relay is kept fixed. As before, the C-RAN downlink sum-rate closely tracks the centralized MIMO sum-rate using a similar amount of fronthaul capacity, and the C-RAN capacity under stochastic geometry scales faster than that under rich scattering.



(a) $\beta = 2.5$, $\lambda_r = 2\lambda_u$. (b) $\beta = 3$, $\lambda_r = 2\lambda_u$.

(c) $\beta = 2.5$, $\lambda_r = \lambda_u^2$. (d) $\beta = 3$, $\lambda_r = \lambda_u^2$.

(e) $\beta = 2.5$, λ_u fixed. (f) $\beta = 3$, λ_u fixed.

Fig. 4: Downlink capacity scaling under stochastic geometry.

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