Noisy Network Coding

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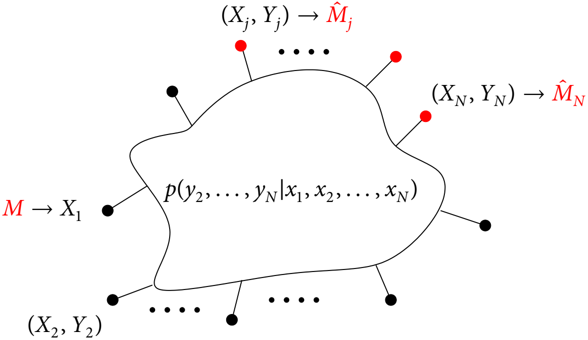
IEEE Information Theory Workshop Cairo 2010

General Noisy Multicast Network



- An N-node discrete memoryless multicast network (DM-MN) $(\mathcal{X}_1 \times \cdots \times \mathcal{X}_N, p(y_2, \ldots, y_N | x_1, \ldots, x_N), \mathcal{Y}_2 \times \cdots \times \mathcal{Y}_N)$
- Source node 1 wishes to send a message M to destination nodes $\mathcal{D} \subseteq [2 : N]$

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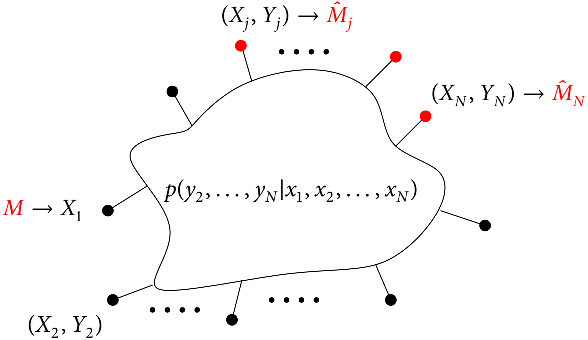

- A $(2^{nR}, n)$ code for the DM-MN:
  - Encoder: $x_1^n(m)$ for each message $m \in [1:2^{nR}]$
  - Relay encoder $j \in [2:N]$: $x_{ji}(y_j^{i-1})$ for each $y_j^{i-1} \in \mathcal{Y}_j^{i-1}$ and $i \in [1:n]$
  - Decoder $k \in \mathcal{D}$: $\hat{m}_k(y_k^n)$ for each $y_k^n \in \mathcal{Y}_k^n$

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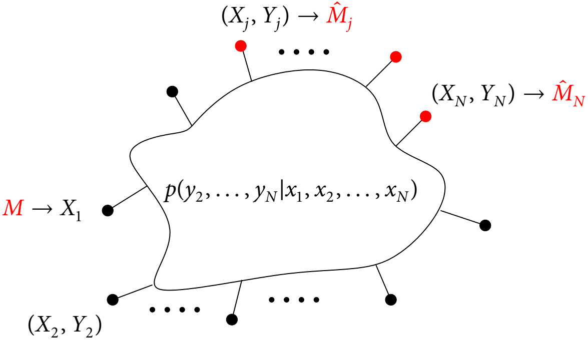

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  - Decoder $k \in \mathcal{D}$: $\hat{m}_k(y_k^n)$ for each $y_k^n \in \mathcal{Y}_k^n$
- The average probability of error $P_e^{(n)} = \mathsf{P}\{\hat{M}_k \neq M \text{ for some } k \in \mathcal{D}\}$

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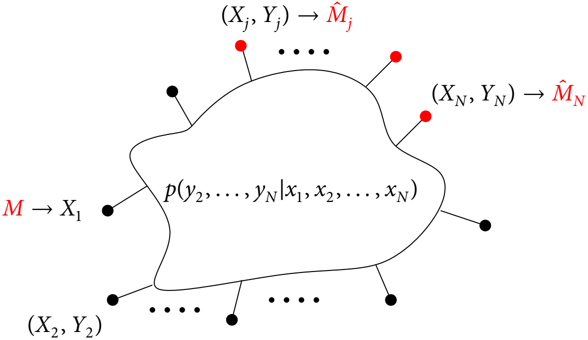

- The rate R is achievable if $\exists$ a sequence of $(2^{nR}, n)$ codes with $P_e^{(n)} \to 0$

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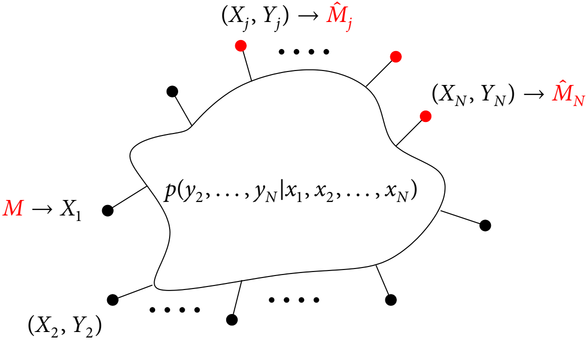

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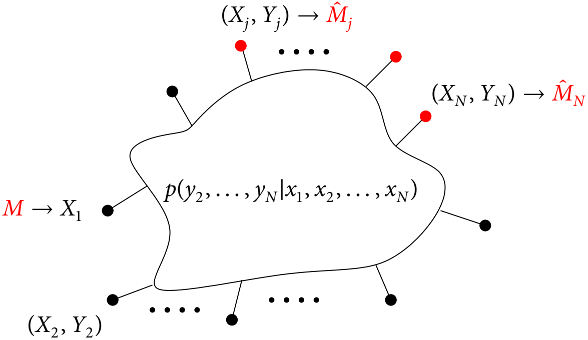

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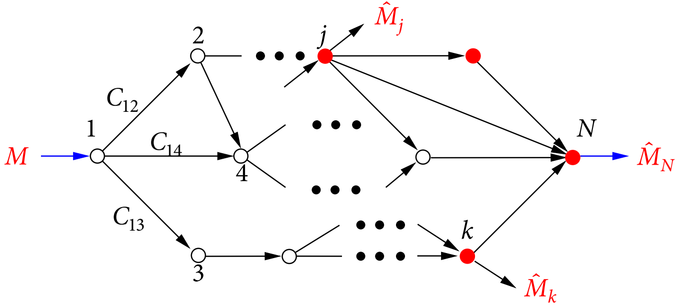
- A computable (single-letter) characterization of the capacity is not known in general

Noiseless Multicast Network

- Noiseless network modeled by the graph $(\mathcal{N}, \mathcal{E}, \mathcal{C})$



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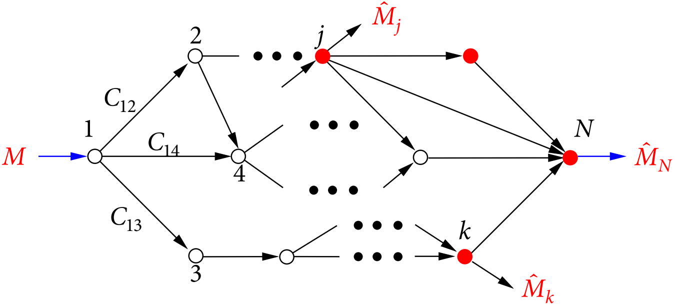

Network coding theorem (Ahlswede–Cai–Li–Yeung 2000)

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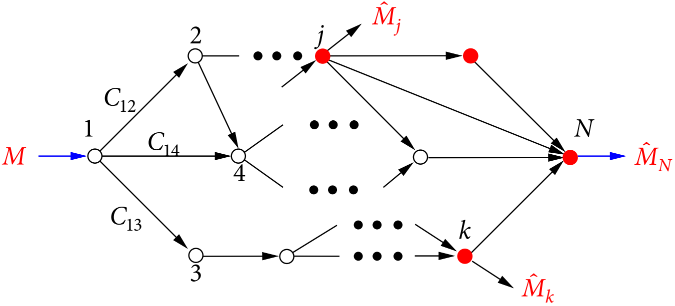

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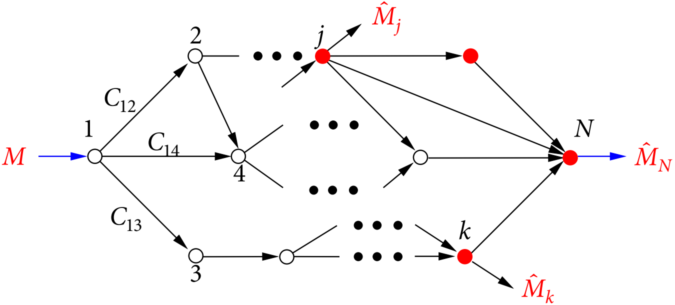

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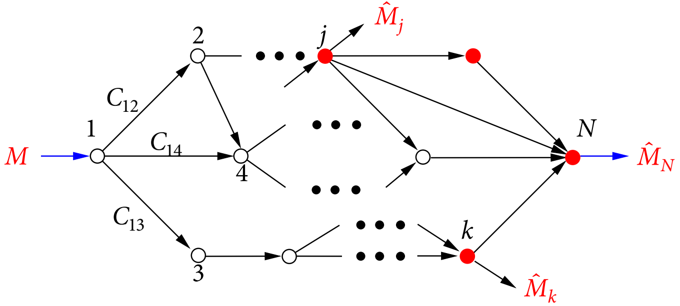

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- In general, network coding is needed (eg. butterfly network)

Extensions

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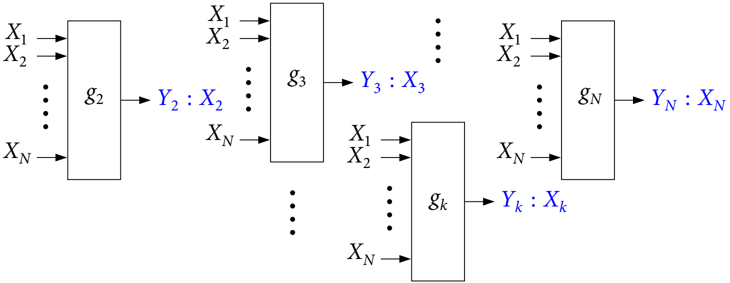

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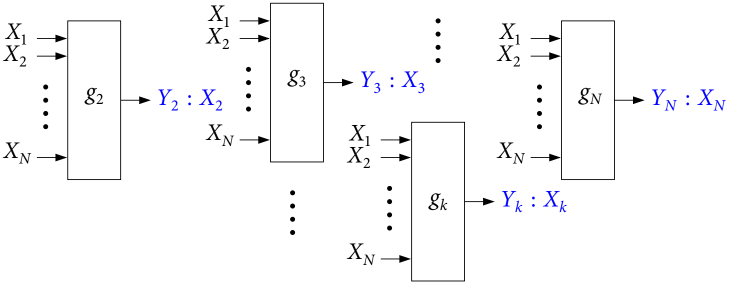

Generalizes noiseless multicast network with broadcast and interference

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  - Finite-field network (Avestimehr–Diggavi–Tse 2007):

    $$Y_k = \sum_{j=1}^N g_{jk} X_j \text{ for } g_{jk}, X_j \in \mathbb{F}_q,\ j, k \in [1:N]$$

    Used to approximate capacity of Gaussian networks in high SNR

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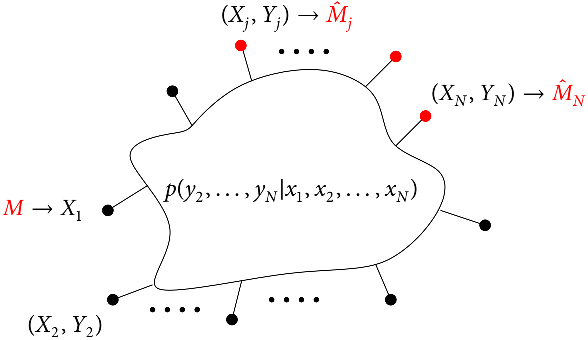
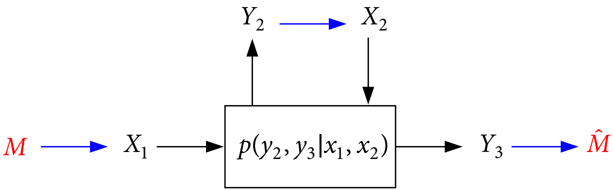


Relay Channel



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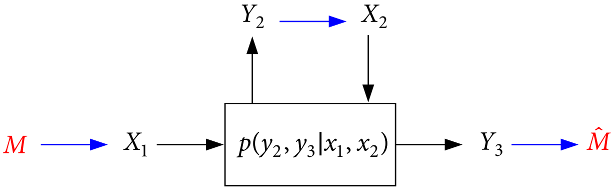

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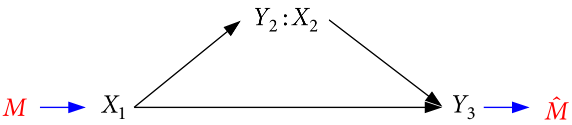

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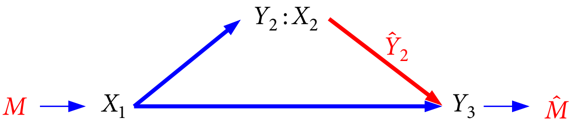

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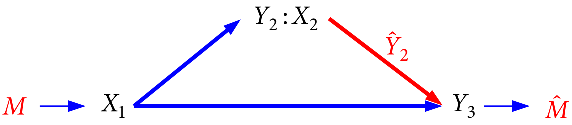

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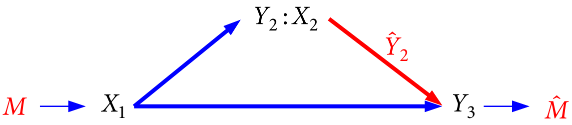

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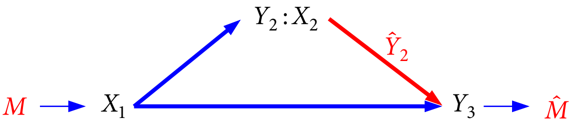

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- Extension to multiple relays (Kramer–Gastpar–Gupta 2005)

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- This particular characterization and corresponding coding scheme (El Gamal–Kim 2009) generalize naturally to networks

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- Network coding is a special case of network compress–forward !!
- Includes as special cases:
  - Network coding for noiseless multicast networks
  - Lower bound on deterministic networks
  - Results on wireless erasure networks
  - Lower bound on semideterministic networks (Lim–Kim–Chung 2009)

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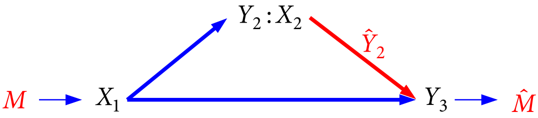

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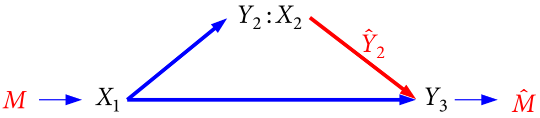

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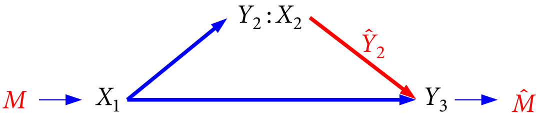

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- The relay uses independently generated compression codebooks:

$$\mathcal{B}_j = \{\hat{y}_2^n(l_j|l_{j-1}) : l_j, l_{j-1} \in [1:2^{nR_2}]\},\ j \in [1:b]$$

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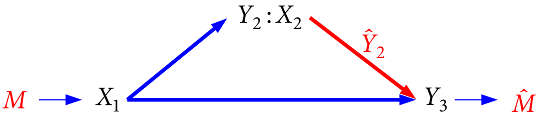
Outline of Proof: Network Compress–Forward

- Use message repetition coding and simultaneous decoding
- For simplicity, consider proof for the relay channel



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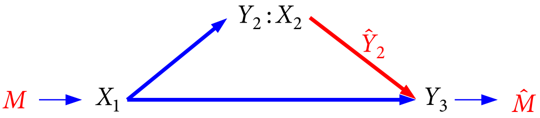

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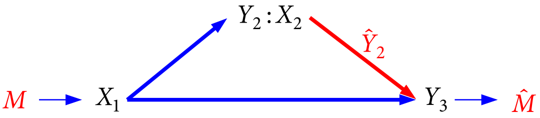

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  Upon receiving $y_2^n(j)$ and knowing $x_2^n(l_{j-1})$, the relay finds jointly typical $\hat{y}_2^n(l_j | l_{j-1})$, and sends $x_2^n(l_j)$ in block $j + 1$

Outline of Proof: Network Compress–Forward

Block	1
X_1	$x_1^n(1, m)$
Y_2	$\hat{y}_2^n(l_1 \| 1)$
X_2	$x_2^n(1)$
Y_3	$\emptyset$

Outline of Proof: Network Compress–Forward

Block	1	2
X_1	$x_1^n(1, m)$	$x_1^n(2, m)$
Y_2	$\hat{y}_2^n(l_1 \mid 1)$	$\hat{y}_2^n(l_2 \mid l_1)$
X_2	$x_2^n(1)$	$x_2^n(l_1)$
Y_3	$\emptyset$	$\emptyset$

Outline of Proof: Network Compress–Forward

Block	1	2	3	…
X_1	$x_1^n(1, m)$	$x_1^n(2, m)$	$x_1^n(3, m)$	…
Y_2	$\hat{y}_2^n(l_1 \mid 1)$	$\hat{y}_2^n(l_2 \mid l_1)$	$\hat{y}_2^n(l_3 \mid l_2)$	…
X_2	$x_2^n(1)$	$x_2^n(l_1)$	$x_2^n(l_2)$	…
Y_3	$\emptyset$	$\emptyset$	$\emptyset$	…

Outline of Proof: Network Compress–Forward

Block	1	2	3	…	$b-1$
X_1	$x_1^n(1, m)$	$x_1^n(2, m)$	$x_1^n(3, m)$	…	$x_1^n(b-1, m)$
Y_2	$\hat{y}_2^n(l_1\|1)$	$\hat{y}_2^n(l_2\|l_1)$	$\hat{y}_2^n(l_3\|l_2)$	…	$\hat{y}_2^n(l_{b-1}\|l_{b-2})$
X_2	$x_2^n(1)$	$x_2^n(l_1)$	$x_2^n(l_2)$	…	$x_2^n(l_{b-2})$
Y_3	$\emptyset$	$\emptyset$	$\emptyset$	…	$\emptyset$

Outline of Proof: Network Compress–Forward

Block	1	2	3	$\ldots$	$b-1$	b
X_1	$x_1^n(1, m)$	$x_1^n(2, m)$	$x_1^n(3, m)$	$\ldots$	$x_1^n(b-1, m)$	$x_1^n(b, m)$
Y_2	$\hat{y}_2^n(l_1\|1)$	$\hat{y}_2^n(l_2\|l_1)$	$\hat{y}_2^n(l_3\|l_2)$	$\ldots$	$\hat{y}_2^n(l_{b-1}\|l_{b-2})$	$\hat{y}_2^n(l_b\|l_{b-1})$
X_2	$x_2^n(1)$	$x_2^n(l_1)$	$x_2^n(l_2)$	$\ldots$	$x_2^n(l_{b-2})$	$x_2^n(l_{b-1})$
Y_3	$\emptyset$	$\emptyset$	$\emptyset$	$\ldots$	$\emptyset$	$\hat{m}$

- Decoding: After receiving $y_3^n(j)$, $j \in [1:2^{nR}]$, the receiver finds unique $\hat{m}$ such that:

$$(x_1^n(j, \hat{m}), \hat{y}_2^n(l_j|l_{j-1}), x_2^n(l_{j-1}), y_3^n(j)) \text{ are jointly typical}$$

  for all $j \in [1:b]$ and for some $l_1, l_2, \ldots, l_b$

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$$\mathsf{P}(\mathcal{E}) \leq \mathsf{P}(\cup_{j=1}^{b} \mathcal{E}_j^c(1,1,1)) + \mathsf{P}(\cup_{m \neq 1} \cup_{l^b} \cap_{j=1}^{b} \mathcal{E}_j(m, l_{j-1}, l_j))$$

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$$\begin{aligned}
\mathsf{P}(\mathcal{E}) &\le \mathsf{P}(\cup_{j=1}^b \mathcal{E}_j^c(1,1,1)) + \mathsf{P}(\cup_{m \ne 1} \cup_{l^b} \cap_{j=1}^b \mathcal{E}_j(m, l_{j-1}, l_j)) \\
&\le \sum_{j=1}^b \mathsf{P}(\mathcal{E}_j^c(1,1,1)) + \sum_{m \ne 1} \sum_{l^b} \prod_{j=2}^b \mathsf{P}(\mathcal{E}_j(m, l_{j-1}, l_j))
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\end{aligned}$$

- If $m \neq 1$ and $l_{j-1} = 1$,

$$\mathsf{P}(\mathcal{E}_j(m, l_{j-1}, l_j)) \le 2^{-n(I(X_1;\hat{Y}_2,Y_3|X_2)-\delta(\epsilon))} =: 2^{-n(I_1-\delta(\epsilon))}$$

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- Similarly, if $m \neq 1$ and $l_{j-1} \neq 1$,

$$\mathsf{P}(\mathcal{E}_j(m, l_{j-1}, l_j)) \le 2^{-n(I(X_1,X_2;Y_3)+I(\hat{Y}_2;X_1,Y_3|X_2)-\delta(\epsilon))} =: 2^{-n(I_2-\delta(\epsilon))}$$

Analysis of the Probability of Error

- Assume $M = 1$, $L_1 = L_2 = \cdots = L_b = 1$
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- If l^{b-1} has k 1s,

$$\prod_{j=2}^b \mathsf{P}(\mathcal{E}_j(m, l_{j-1}, l_j)) \le 2^{-n\left(kI_1+(b-1-k)I_2-(b-1)\delta(\epsilon)\right)}$$

Analysis of the Probability of Error

$$\sum_{m \neq 1} \sum_{l_b} \sum_{l^{b-1}} \prod_{j=2}^{b} \mathsf{P}(\mathcal{E}_j(m, l_{j-1}, l_j))$$

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$$\sum_{m\neq 1}\sum_{l_b}\sum_{l^{b-1}}\prod_{j=2}^{b}\mathsf{P}(\mathcal{E}_j(m, l_{j-1}, l_j))$$

$$\leq \quad \sum_{m\neq 1}\sum_{l_b}\sum_{k=0}^{b-1}\binom{b-1}{k}2^{n(b-1-k)R_2}\cdot 2^{-n\left(kI_1+(b-1-k)I_2-(b-1)\delta(\epsilon)\right)}$$

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$$\leq \sum_{m\neq 1}\sum_{l_b}\sum_{k=0}^{b-1}\binom{b-1}{k}2^{-n\left((b-1)(\min\{I_1,I_2-R_2\}-\delta(\epsilon))\right)}$$

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&= \sum_{m\neq 1}\sum_{l_b}\sum_{k=0}^{b-1}\binom{b-1}{k}2^{-n\left(kI_1+(b-1-k)(I_2-R_2)-(b-1)\delta(\epsilon)\right)}\\
&\leq \sum_{m\neq 1}\sum_{l_b}\sum_{k=0}^{b-1}\binom{b-1}{k}2^{-n\left((b-1)(\min\{I_1,\, I_2-R_2\}-\delta(\epsilon))\right)}\\
&\leq 2^{b+n\left(bR+R_2-(b-1)(\min\{I_1,\, I_2-R_2\}-\delta(\epsilon))\right)},
\end{aligned}$$

Analysis of the Probability of Error

$$
\begin{aligned}
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which $\to 0$ as $n\to\infty$ if $R<\frac{b-1}{b}(\min\{I_1,\ I_2-R_2\}-\delta'(\epsilon))-\frac{R_2}{b}$

Analysis of the Probability of Error

$$\sum_{m\neq 1}\sum_{l_b}\sum_{l^{b-1}}\prod_{j=2}^{b}\mathsf{P}(\mathcal{E}_j(m, l_{j-1}, l_j))$$

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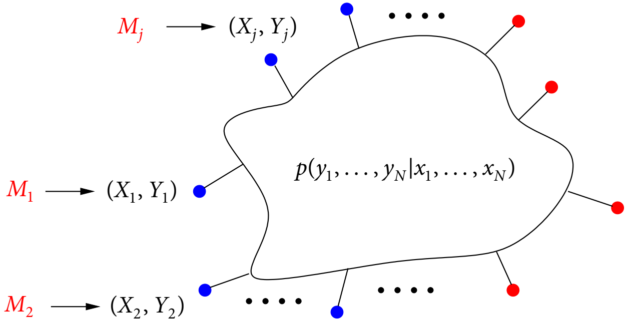
- Finally, by eliminating $R_2 > I(\hat{Y}_2; Y_2|X_2)+\delta(\epsilon')$ and letting $b\to\infty$,

  $R < \min\{I(X_1; \hat{Y}_2, Y_3|X_2),\ I(X_1, X_2; Y_3) - I(\hat{Y}_2; Y_2|X_1, X_2, Y_3)\} - \delta'(\epsilon) - \delta(\epsilon')$

Noisy Multi-source Multicast Network



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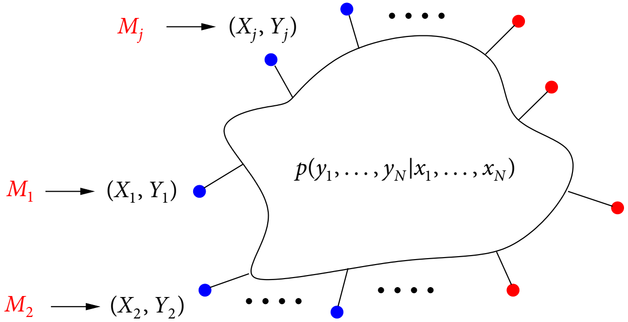


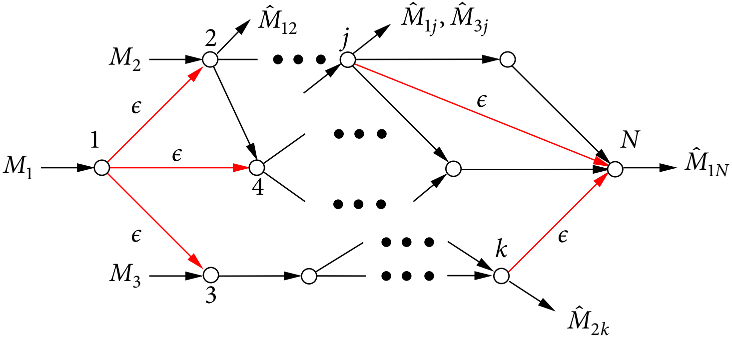
Theorem (Noisy network coding inner bound)

$(R_1, \ldots, R_N)$ achievable if

$$\sum_{j \in \mathcal{S}} R_j < \min_{k \in \mathcal{S} \cap \mathcal{D}} I(X(\mathcal{S}); \hat{Y}(\mathcal{S}^c), Y_k | X(\mathcal{S}^c)) - I(Y(\mathcal{S}); \hat{Y}(\mathcal{S}) | X^N, \hat{Y}(\mathcal{S}^c), Y_k)$$

for all cutsets $\mathcal{S}$ for some $\prod_{j=1}^N p(x_j) p(\hat{y}_j | y_j, x_j)$

Example: Erasure Network



- Link failure is observed as an erasure symbol

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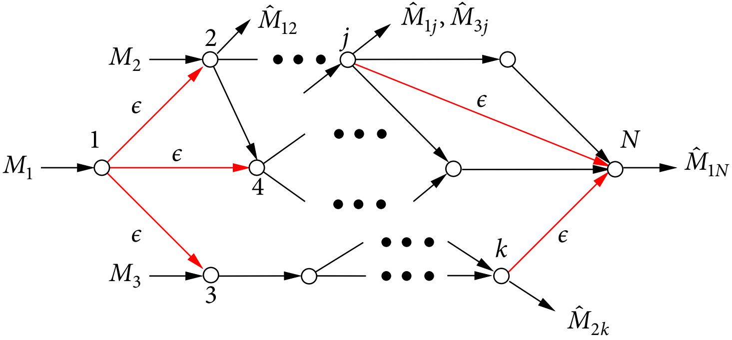

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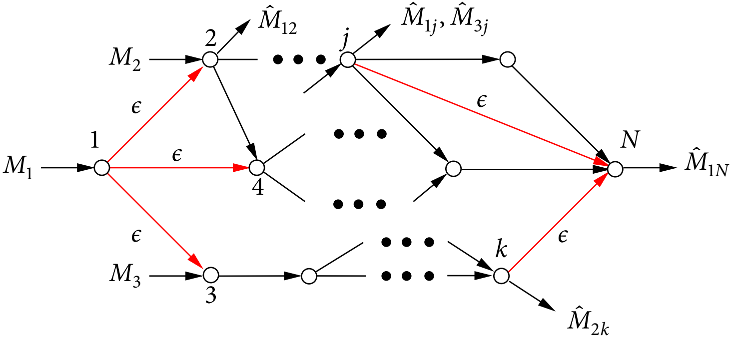

- Link failure is observed as an erasure symbol
- Assume destination nodes have access to network erasure pattern
- We recover multi-source multicast capacity region (Dana et al. 2006):

$$\sum_{j \in \mathcal{S}} R_j \leq \sum_{j \in \mathcal{S}} C_{jk}(1 - \text{P}\{\text{link } (j,k) \text{ is erased for all } k \in \mathcal{S}^c\})$$

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- Channel model: $Y^N = GX^N + Z^N$
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- With $\hat{Y}_j = Y_j + \hat{Z}_j$, $\hat{Z}_j \sim \mathrm{N}(0,1)$, noisy network coding bound becomes

$$\sum_{j\in\mathcal{S}} R_j < \frac{1}{2}\log\left|I + \frac{P}{2}G(\mathcal{S})G(\mathcal{S})^T\right| - \frac{|\mathcal{S}|}{2}$$

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- On the other hand, the cutset bound becomes

$$\sum_{j \in \mathcal{S}} R_j \leq \frac{1}{2} \log \left| I + \frac{P}{2} G(\mathcal{S}) G(\mathcal{S})^T \right| + \frac{N}{2} \log(2N)$$

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- With $\hat{Y}_j = Y_j + \hat{Z}_j$, $\hat{Z}_j \sim \mathrm{N}(0,1)$, noisy network coding bound becomes

$$\sum_{j \in \mathcal{S}} R_j < \frac{1}{2} \log \left| I + \frac{P}{2} G(\mathcal{S}) G(\mathcal{S})^T \right| - \frac{|\mathcal{S}|}{2}$$

- On the other hand, the cutset bound becomes

$$\sum_{j \in \mathcal{S}} R_j \le \frac{1}{2} \log \left| I + \frac{P}{2} G(\mathcal{S}) G(\mathcal{S})^T \right| + \frac{N}{2} \log(2N)$$

- Thus, compress–forward is optimal within $(N/2)\log(4N)$ bits uniformly

Example: Gaussian Network

- Channel model: $Y^N = GX^N + Z^N$
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- This generalizes single-source result by Avestimehr–Diggavi–Tse (2007)

Summary

Forwarding

Noiseless Unicast Networks

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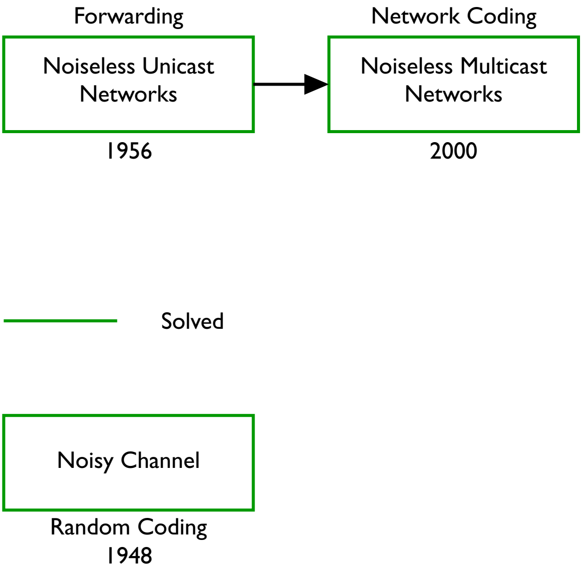
Solved

Noisy Channel

Random Coding

1948

Summary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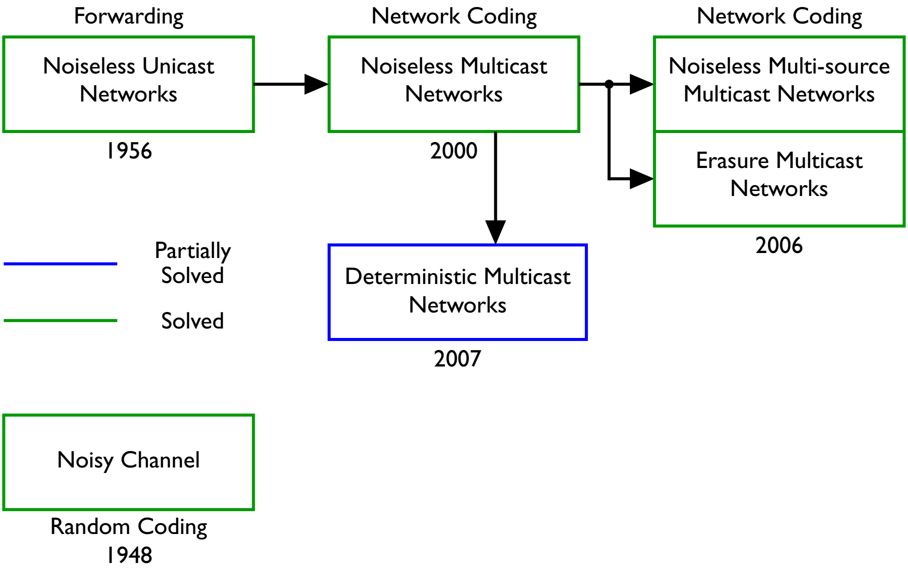

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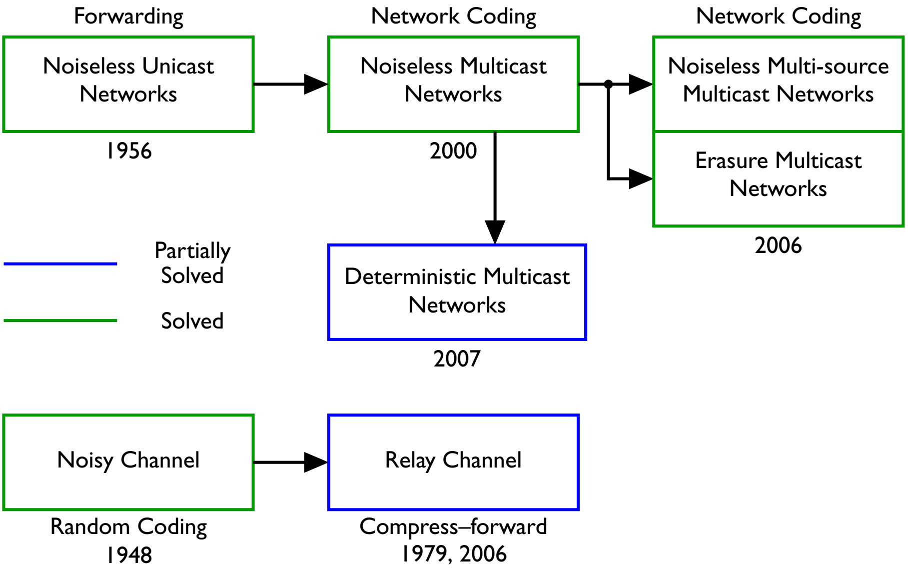

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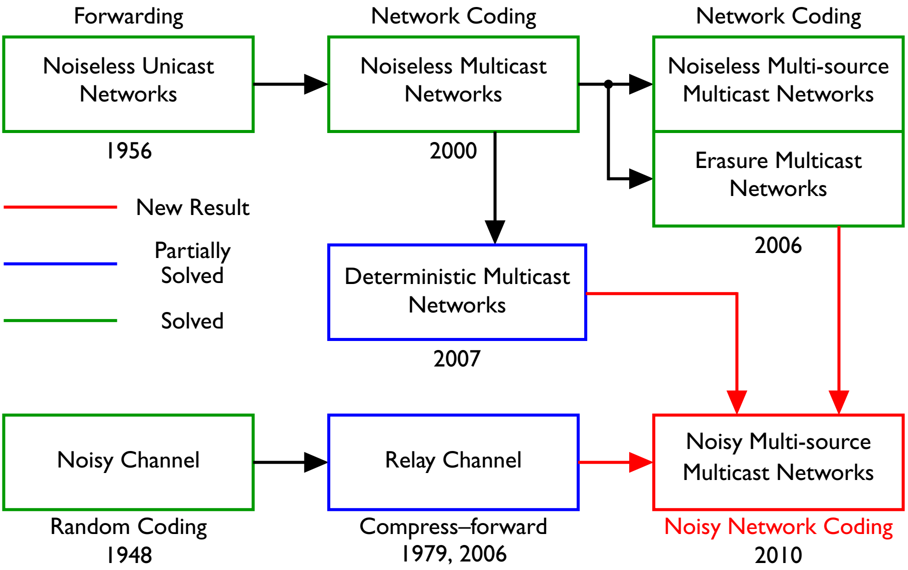
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Concluding Remarks

- Network compress forward = Noisy network coding
  - Recent approaches (bottom-up): Generalized network coding to deterministic and erasure networks
  - By comparison, noisy network coding gives a top-down approach for a general network and combines network coding and compress–forward, including all previous results as special cases
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- Can be further combined with decode–forward schemes (Cover–El Gamal 1979 Thm. 7, Kramer–Gastpar–Gupta 2005 Thm. 5)