More on Lab #7

CMOS Inverter:

NMOS is "on" when Vin = VDD (and PMOS is "off")
PMOS is "on" when Vin = 0 (and NMOS is "off")

Cin is gate capacitance and other parasitics
Cout is drain capacitance and other parasitics

\[ R_0 = \frac{1}{2}(R_{p} + R_{n}) \] average of PMOS and NMOS on resistance
\[ C_{out} = C_{in} + C_{out} \] each inverter sees its own output cap, and input of next stage

\[ T_0 = \frac{1}{2}(T_{DH} + T_{DL}) \] average of low-high and high-low transitions
\[ T_{DLH} = 0.69 R_{W} C_{out} \] \( e^{-\frac{t}{RC}} = 0.5 \) \( t = RC \ln 0.5 = 0.69 RC \)
\[ T_{DLH} = 0.69 R_{p} C_{out} \]

Tin = 0.69 R0 Cin: last stage sees input of buffer too

Total period of oscillator = \( 2N T_0 + 2T_{in} \)

Factor of 2 from 1\( \rightarrow \)0 and 0\( \rightarrow \)1 transitions
Single Inverter

\[ V_{in} \quad \xrightarrow{D} \quad V_{out} \]

\[ V_{in} \quad \xrightarrow{R_0} \quad V_{out} \]

\[ \frac{V_{out}}{V_{in}} = \frac{-A_i}{1 + s \cdot \tau} \to \tau = R \cdot C \]

Cascade of three inverters

\[ \frac{-A_i}{1 + s \cdot \tau} \]

\[ \frac{-A_i}{1 + s \cdot \tau} \]

\[ \frac{-A_i}{1 + s \cdot \tau} \]

\[ B = 1 \]

\[ AB = \frac{A_i^3}{(1 + s \cdot \tau)^3} \]

\[ AB = \frac{A_i^3}{1 + 3s \cdot \tau + 3s^2 \cdot \tau^2 + s^3 \cdot \tau^3} \]

where does it cross the real axis?

Imaginary part is \( 3s \cdot \tau + s^3 \cdot \tau^3 \)

(odd factors of \( s = j \omega \))

\[ 3 + s^2 \cdot \tau^2 = 0 \]

\[ 3 = \omega^2 \cdot \tau^2 \quad \rightarrow \quad \omega = \frac{\sqrt{3}}{\tau} \quad \rightarrow \quad \text{plug back into } AB \]

\[ AB = \frac{A_i^3}{1 - 3\omega^2 \cdot \tau^2} = \frac{A_i^3}{1 - 3 \left( \frac{3}{\omega} \right)^2 \cdot \tau^2} = \frac{-A_i^3}{8} \]

Ring will oscillate if \( A_i^3 > 8 \)

or \( A_i > 2 \)
Quartz oscillators

used for very accurate clocks \( \rightarrow \) most watches and computers
important for synchronizing data transmission in communications

\( \text{SiO}_2 \) quartz

\[ f = \frac{1}{2\pi \sqrt{LC}} \]

A good quartz watch is accurate to \( \pm 15 \text{ s/year} \), \( \Rightarrow Q \approx 10^6 \)

works based on piezoelectric effect:
- applied electric field squeezes and distorts the crystal
- can be modeled as RLC circuit
  \[ \frac{1}{2} kx \leftrightarrow \frac{1}{2} m \frac{d^2x}{dt^2} \leftrightarrow \frac{1}{2} C \frac{d^2V}{dt^2} \]

Example oscillator circuit: (there are lots of other designs)

Forces: \( V_{out} \approx V_{in} \) at DC

Inverter works like an amplifier

Use same procedure as last example to find where Nyquist plot crosses real axis
Relaxation Oscillator:

Schmitt trigger:

$$V_+ = \frac{V_{in}}{R_1} + \frac{V_{+} - V_{supply}}{R_2} = 0$$

$$V_+ = 0$$

$$V_{in} = -\frac{R_1}{R_2} V_{supply} \Rightarrow \text{transition voltages are at } \pm \frac{R_1}{R_2} V_{supply}$$

Integration:

$$V_+ = \frac{1}{RC} \int V_{in} dt$$

$$V_+ = 0$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{SCR}$$

Laplace transform: $$\frac{1}{s} F(s) \leftrightarrow \int_0^t t e^x dx$$

when $$V_{in} = -V_{supply},$$

$$V_{out}(t) = -\frac{1}{CR} \int_0^t V_{supply} \, dx = \frac{V_{supply}}{CR} t$$

Transitions at $$V_{supply} \frac{t}{CR} = \frac{R_1}{R_2} V_{supply}$$

$$t = CR \frac{R_1}{R_2} = \frac{1}{4} \text{ period}$$