Lab #2 Discussion

Purpose: Pass voice signals (<4kHz) but reject higher frequency DSL signals (>4kHz - 4MHz)

First - try simple Butterworth filter (2nd order)

Note - rule of -20dB/decade per pole
is same as -6dB/ octave per pole
4kHz -> 32kHz is 3 octaves

3 x 6 x 2 poles => 36dB...should be fine. (?)

However, w = \omega_0 is the -3dB point \left| H(f) \right|^2 = \frac{1}{1 + (\frac{f}{\omega_0})^2n}
We need -1dB at 4kHz, so \omega_0 must be > 4kHz
We do not have 3 full octaves available.
2nd order Butterworth:

\[ |H(\omega)|^2 = \frac{1}{1 + (\frac{\omega}{\omega_0})^2} \]

\[ |H(\omega)|^2 = -1 \text{dB at } 4 \text{ kHz} \]

\[ 10^{-0.1} = 0.7943 = \frac{1}{1 + (\frac{4 \times 10^3}{\omega_0})^2} \]

\[ \omega_0 = 5.61 \text{ kHz} \]

Next check -30dB spec at 32 kHz

\[ |H(\omega)|^2 = \frac{1}{1 + (\frac{\omega}{\omega_0})^2} = \frac{1}{1 + (\frac{32}{5.61})^2} = 9.4 \times 10^{-4} \]

(note dB is 20 log10 Gain or 10 log10 Gain^2)

\[ 10 \log_{10} (9.4 \times 10^{-4}) = -30.26 \text{ dB} \]

Just barely meets both specs, by a fraction of a dB.

This is not enough for typical device tolerances.

Electronic devices have values based on "preferred numbers."

Divide \(1\) to \(10\) into \(6, 12, 24, 48, 96\) steps.

These become \(20\%, 10\%, 5\%, 2\%, 1\%\) tolerances.

Equal ratio between adjacent steps: \(X^n = 10 \rightarrow X = 10^{\frac{1}{n}}\)

Ex: \(n = 6\)

\[ X = 1.4678 \]

\[ X^n |_{m=0} = 1, 1.47, 2.15, 3.16, 4.64, 6.81, 10 \]

Standard 20% choices: 10, 15, 22, 33, 47, 68, 100 ...
How to have better performance margin:

1. Use 3rd order filter → 50% more expensive
2. Use something other than Butterworth.

How do we design this filter?

\[ H(s) = \frac{(0.7943)^2}{1 + \frac{2s}{\omega_0} + \frac{s^2}{\omega_0^2}} \]

Find \( s \) and \( \omega_0 \)

DC gain = \(-1d\)B

\[ \text{remember } \frac{d}{ds} \left( \frac{1}{|H(s)|^2} \right) = 0 \]

\[ \frac{|H(s)|^2}{|H(j\omega)|^2} = \frac{1}{4\omega^2(1-\omega^2)^2} = 10^{-21} = 1.25 \times 10^{-21} \]

Solve for \( \omega \). Optimum value in range of 0.1-0.7

Next solve for \( \omega_0 \)

\[ |H(\omega)|^2 = \frac{0.7943}{(1-\frac{\omega^2}{\omega_0^2})^2 + (2\frac{\omega}{\omega_0})^2} \]
Optimize values of C and use to have wide margin on all specs

Why are we using an active filter instead of RLC?

Inductors are:
- Lossy (low Q)
- Large (especially on-chip)
- Have low resonance frequency
- Less accurate than R, C
- More expensive (often wire wound)

Instead we use Sallen-Key active filter

(R.P. Sallen, E.L. Key, MIT Lincoln Lab, 1955)
Op-amp with negative feedback: $V_+ = V_- = V_{out}$

KCL at node $X$:

$$\frac{V_x - V_{in}}{Z_1} + \frac{V_x - V_{out}}{Z_3} + \frac{V_x - V_+}{Z_2} = 0$$

KCL at node $+$: (equal to $V_{out}$)

$$\frac{V_{out} - V_x}{Z_2} + \frac{V_{out}}{Z_4} = 0$$

$$V_x = V_{out} \left( \frac{Z_2}{Z_4} + 1 \right)$$

Substitute into this equation and rearrange

$$\frac{V_{out}}{V_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

or

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{Z_1 + Z_2}{Z_4} + \frac{Z_1 Z_2}{Z_3 Z_4}}$$

For our case:

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + s C_1 (R_1 + R_2) + s^2 C_2 R_1 R_2}$$

$$= \frac{1}{1 + \frac{2 \pi f}{\omega_0} + \frac{s^2}{\omega_0^2}}$$
Note that we can also make a high-pass filter:

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_1 R_2}{\frac{1}{s^2 C_1 C_2} + R_1 \left( \frac{1}{s C_1} + \frac{1}{s C_2} \right) + R_1 R_2}
\]

\[
= \frac{s^2 C_1 C_2 R_1 R_2}{1 + R_1 (s C_1 + s C_2) + s^2 C_1 C_2 R_1 R_2}
\]

Other notes on lab #2:

Reducing DC gain:

Output resistance:

\[
A(V - V_0) = V_t
\]