Input Impedance

\[ B = \frac{R_1}{R_1 + R_2} \]

\[ I_{in} = \frac{V_{in} - B V_{out}}{R_d} = \frac{V_{in}}{R_d} (1 - BH) = \frac{V_{in}}{R_d} (1 - \frac{AB}{1 + AB}) \frac{V_{in}}{R_d} (1 + AB) \]

\[ \frac{I_{in}}{V_{in}} = \frac{1}{R_d} \left( 1 + \frac{GB}{3} \right) = R_d + \frac{1}{s C_{eff}} \quad C_{eff} = \frac{1}{GB R_d} \]

Input impedance is \( R_d \) in series with small capacitor.

E.g. \( G = 2 \times 10^6 \), \( B = 0.1 \), \( R_d = 10^7 \rightarrow 0.16 \text{ pF} \)

\[ Z = R + j X \]

at \( w = GB \), \( X_{eff} = \frac{GB R_d}{GB} = R_d \)

for \( w < GB \), \( X_{eff} > R_d \rightarrow \text{input is capacitive} \)

for \( w > GB \), \( X_{eff} < R_d \rightarrow \text{input is resistive} \)
Output Impedance

\[ V_{out} = AV_{in} = \frac{A}{1 + AB} \]
\[ I_{sc} = \frac{AV_{in}}{R_{o}} \]
\[ Z_{out} = \frac{V_{in}}{I_{sc}} = R_{o} \frac{A}{AV_{in}/R_{o}} = \frac{R_{o}}{1 + AB} = \frac{1}{A} \]

\[ Z_{out} = \frac{R_{o}}{1 + GB} = \frac{1}{R_{o} + \frac{GB}{R_{o}}} \]

Output impedance is \( R_{o} \) in parallel with inductor \( L_{eff} = \frac{R_{o}}{GB} \)

for \( w = GB \), \( X_{eff} = R_{o} \)

for \( w < GB \), \( X_{eff} < R_{o} \rightarrow \) output is inductive

for \( w > GB \), \( X_{eff} > R_{o} \rightarrow \) output is resistive

3 effects of feedback

1. Reduces and stabilizes gain, increasing bandwidth
2. Increases \( Z_{in} \), but makes it capacitive
3. Reduces \( Z_{out} \), but makes it inductive
Slew Rate Limiting

\[ i = C \frac{dV_{at}}{dt} \]

\[ V_{out} = A \sin(cut) \]

\[ i = ACw \cos(wt) \]

\[ |i| = ACw \rightarrow \text{current scales with:} \]
  - Signal amplitude
  - Capacitance
  - Frequency

\[ \frac{dV_{at}}{dt} \frac{1}{|i_{max}|} = \frac{i_{max}}{C} \]

Overshoot and Rise Time

\[ \% \text{ overshoot} = \left( \frac{v}{x} - 1 \right) \times 100 \]

10% to 90% rise time
Stability Theory

Transfer function always has form $H(s) = \frac{A}{1+AB}$
Always has $1+AB$ in denominator
Numerator depends on how the input enters the circuit

\[ \frac{A}{1+AB} \]

Another common form:

\[ \frac{AB}{1+AB} \]

If we cut the feedback loop here,

Then we have \( Out = AB In \) where \( AB \) is the open-loop transfer function.

We can analyze \( AB \) to determine stability of the closed loop system.

Circuit cannot be stable if any subcircuit \((A,B)\) is unstable.

How to determine stability:

Does \( 1+AB \) have any zeros in right half plane?

Is \( AB = -1 \) for any \( s \)?

Remember \(-1 = 1 \times e^{-180°}\)

Point of neutral stability (just barely stable/unstable):

\[ |AB| = 1 \text{ and } <AB = ±180° \]
Gain and Phase Margin

Gain Margin = \(-20 \log_{10} |AB|\) at \(\omega\) where \(<AB> = -180^\circ\)
Phase Margin = \(<AB> + 180^\circ\) at \(\omega\) where \(|AB| = 1\) (0 dB)

Example: \(AB = \frac{K}{(s+1)(s+10)(s+100)}\) for \(K = 10^4, 10^5, 10^6\)

<table>
<thead>
<tr>
<th>(K)</th>
<th>(GM)</th>
<th>(PM)</th>
<th>Stability</th>
<th>Roots of ((s+1)(s+10)(s+100) + K = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^4)</td>
<td>20 dB</td>
<td>45°</td>
<td>stable</td>
<td>(-101, -5 \pm 9j)</td>
</tr>
<tr>
<td>(10^5)</td>
<td>0 dB</td>
<td>0°</td>
<td>marginally stable</td>
<td>(-109, -0.8 \pm 30j)</td>
</tr>
<tr>
<td>(10^6)</td>
<td>-20 dB</td>
<td>45°</td>
<td>unstable</td>
<td>(-148, 18 \pm 80j)</td>
</tr>
</tbody>
</table>

\(\omega = 32\)
Nyquist Stability Criterion

Consider a contour $C$ in the $s$ plane.

If $C$ does not enclose any poles or zeros, it will not go through a full 360° of phase.

$$1 + AB = \frac{\frac{\pi}{\pi} (s - z)}{\frac{\pi}{\pi} (s - p)} = \text{Mag} e^{i\Phi}$$

$$\text{Mag} = \frac{\pi}{\pi} \text{magnitude from zeros}$$
$$\Phi = \text{Z phase from zeros} - \text{Z phase from poles}$$

We take a contour along imaginary axis and extending to $\infty$ on positive real side.

Total number of times contour of $1 + AB$ encircles origin: $N = Z - P$ (clockwise)

$$Z = \# \text{Zeros of } 1 + AB \text{ in RHP}$$
$$P = \# \text{Poles of } 1 + AB \text{ in RHP}$$

But poles of $1 + AB = \text{poles of } AB$

We know these have no poles in RHP if $A$ and $B$ are stable.

If contour of $1 + AB$ encircles origin clockwise, system is unstable.

We actually look for encirclements of $-1$ in plot of $AB$.

If $AB = -1$, there is a pole right on the $w$ axis.