Lab #4: Design of a Differentiator

Laplace transform: \( F(s) \leftrightarrow f(t) \), \( f(0) \leftrightarrow \) initial value

We need a system where \( H(s) = \frac{5 \pi}{s} \leftrightarrow \) scale factor

**Problems with this system:**

- Non causal: can’t anticipate phase of incoming signal at all frequencies
- Can’t have Gain \( \rightarrow \infty \) for \( \omega \rightarrow \infty \)
  (can’t have more zeros than poles)

Consider this circuit:

**Simple model:** \( V_2 = 0 \)

\[
0 - V_{in} \frac{Z_i}{Z_i} + 0 - V_{out} \frac{Z_e}{Z_e} = 0
\]

\[
\frac{V_{out}}{V_{in}} = -\frac{Z_e}{Z_i}
\]

\[
\frac{V_{out}}{V_{in}} = -5 \pi CR
\]

or \( V_{out}(s) = -5 \pi V_{in}(s) \)

\[
\frac{V_{out}(t)}{V_{in}(t)} = -7 \frac{d V_{in}(t)}{dt}
\]

\( \tau = RC \)

note RC has units of time
What about \( x = RC \) factor?

Example: \( RC = 10^{-3} \)

\[
\begin{align*}
\frac{dv}{dt} &= \frac{2v}{2ms} - 10^{-3}y_s + v \\
\frac{dv}{dt} &= -RCv - v \\
\end{align*}
\]

However, what you actually see in the lab is:

This must be a second-order system.

More detailed analysis:

\[
\begin{align*}
V_{out} &= -AV_i \\
V_i &= \frac{V_{out}}{Z_f} + \frac{V_i}{Z_i} \\
V_i (Z_f + Z_i) &= \frac{V_{out}}{Z_f} + \frac{V_i}{Z_i} \\
V_i (Z_i + Z_f) &= V_{out}Z_i + V_inZ_f \\
V_i &= V_{out}Z_i + V_inZ_f \\
V_{out} &= -AV_i = -A\left(\frac{Z_i}{Z_i+Z_f} + \frac{V_i}{Z_i+Z_f}\right) \\
V_{out} (1 + \frac{Z_i}{Z_i+Z_f}) &= -AV_i \frac{Z_i}{Z_i+Z_f} \\
\frac{V_{out}}{V_i} &= -\frac{Z_f}{Z_i} \quad \text{where } B = \frac{Z_i}{Z_i+Z_f} \\
H_{ideal} &= -\frac{Z_f}{Z_i} \quad \text{and } H_{real} = H_{ideal} \cdot \frac{AB}{1+AB}
\end{align*}
\]
Loop gain $AB$ for Differentiator

Phase margin approaches zero
- response will be $\infty$ with overshoot and oscillations
- may even be unstable (considering higher op-amp poles)

How do we eliminate this? - Compensation (next lecture)

Remainder of this lecture - review for Midterm #1